

Electricity Demand Forecast Model Review (DRAFT)

Background

Electricity demand forecasts have been presented by a number of agencies in the past. Part F of the Electricity Governance Rules 2003 requires the Electricity Commission to publish demand forecasts as part of the centralised dataset. This paper outlines the process used to review and select the model to be used for the Electricity Commission's forecasts.

Process

Transpower prepares demand forecasts on an annual basis for their asset valuation process and for long term network planning. The forecasts have previously been published as part of the System Security Forecast and in the Transpower ODV Valuation report. A copy of Transpower's model was provided to the Electricity Commission and has been used as a starting point for building the software used to assess the alternative models.

The high level approach has been to:

- replicate Transpower's existing model in an appropriate software package;
- construct an environment to assess modelling uncertainty;
- establish a set of alternative models;
- review and refine the alternative models;
- select the preferred model.

Key Drivers

There is a wide range of potential drivers of long term electricity demand, ranging from immigration rates to demand for New Zealand produced goods and long term weather trends. The drivers can be split into 4 broad areas: economic activity (measured by GDP), demographics, electricity prices (and demand responsiveness), and energy intensity (determined by the type of electricity end use and technology). The availability of reliable series of historical and forecast data largely determines the drivers that can be utilised for long term forecasting. Appendix A contains a list of the key drivers and contributing factors relevant to grid related demand.

The models assessed in this analysis are focused at producing forecasts that reflect changes in historical demand and its drivers. Underlying historical improvements in energy efficiency for example are already reflected in the demand numbers. Possible step changes in demand may occur as a result of policy changes. It is outside the scope of this analysis to consider the impact of future policy changes and if, and how, they should be wound into the demand forecasts.

General Modelling

The scope of this review is limited to forecasting electricity demand. Models capable of modelling combinations of energy types such as multi-industry general or partial equilibrium models will be considered as part of a separate process.

Different sources of electricity demand have different growth characteristics. Growth in demand sourced from the basic metals industry for example has different drivers to growth associated with domestic residences. The ability to model specific areas of demand is reliant on the availability of relevant historical and forecast data. Although detailed breakdowns of demand are often available on a year by year basis, consistent data over sufficiently long periods is generally available only at a largely consolidated level. While there may be scope for additional analysis in the future, demand modelling considered in this review has been focused at three key demand groups:

- Residential
- Commercial and Light Industrial
- Heavy Industrial

Forecasts are at grid exit point. Therefore they include local lines losses, but exclude consumption that is met by generation embedded in the local lines networks.

Model type

Transpower's model uses an econometric approach to forecasting. In brief, this involves assessing the relationship between historical demand and likely key drivers of demand (such as GDP and population), then using this relationship to forecast future demand using forecasts of the key drivers. These are often referred to as regression based models.

An alternative modelling approach is time series forecasting. This approach uses a detailed analysis of patterns in historical demand to produce a forecast of future demand.

There are also alternative approaches available such as neural network and hybrid models. These typically use multiple techniques and inputs, and produce forecasts based on the mix that produces the best results given the input data available at the time.

Time series models are useful for short term forecasting and for developing a picture of underlying patterns in data (hydrology patterns and changes in half hourly load patterns at individual points of supply are good examples). However they are of limited use for long term forecasts in some situations, such as where there are underlying changes in the key drivers of demand as is being considered here.

Hybrid and neural network models have the potential to produce forecasts that perform well compared to the more traditional modelling approaches. Their main disadvantage is their "black box" nature. Given that the forecasts will be made publicly available through the centralised data set, and will face public scrutiny through consent processes, the forecasting model needs to be intuitive and easily explained to "non-experts". At this stage we believe that neural networks and the like do not meet these criteria, although they may be useful as a validation tool for the forecasts. The use of such models may be assessed in more detail at a later date.

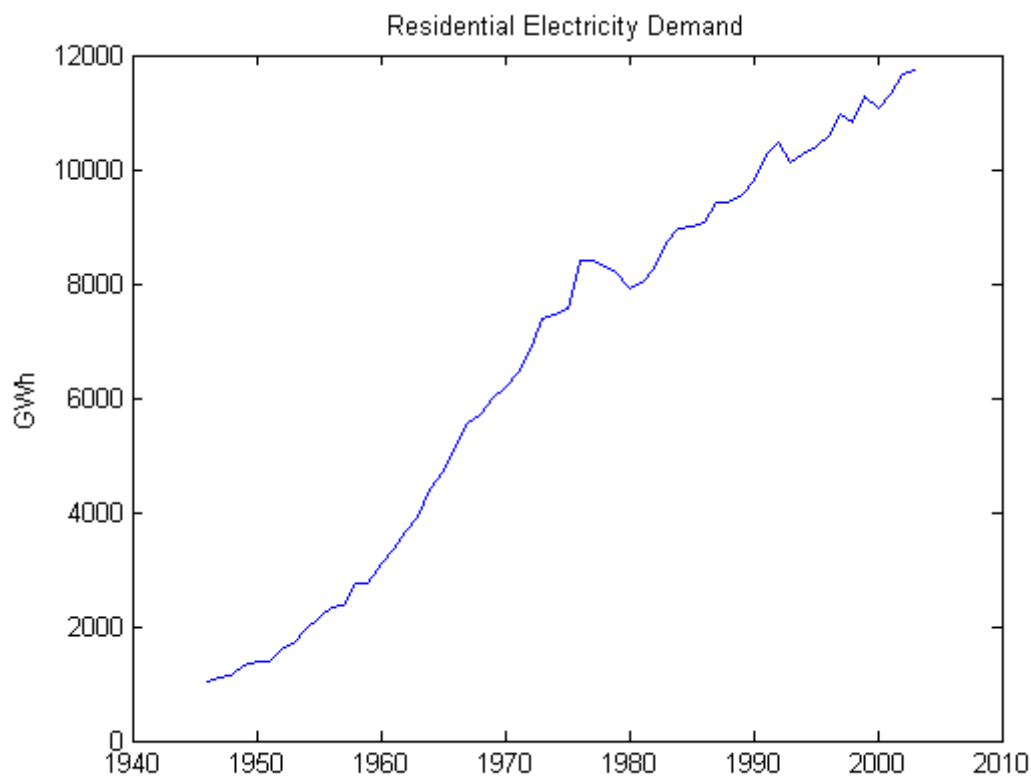
For the reasons noted above, the assessment of alternative models carried out as part of this process has been restricted to those using an econometric approach.

Software environment

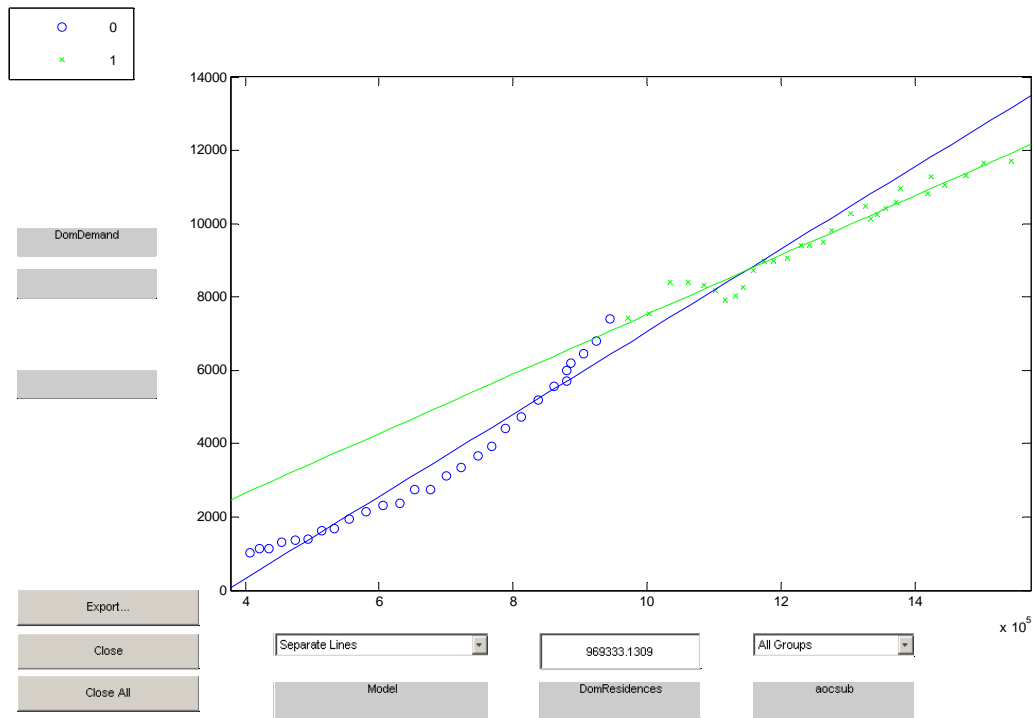
A key focus of the review was on assessing uncertainty in the forecasts rather than just producing a spot forecast of demand. The MATLAB software package was selected as a platform for the analysis due to its scripting flexibility and the relative ease of setting up multiple runs of the demand models. Once a preferred model has been selected it is intended to rebuild it in a more widespread application such as Excel.

Modelling Period

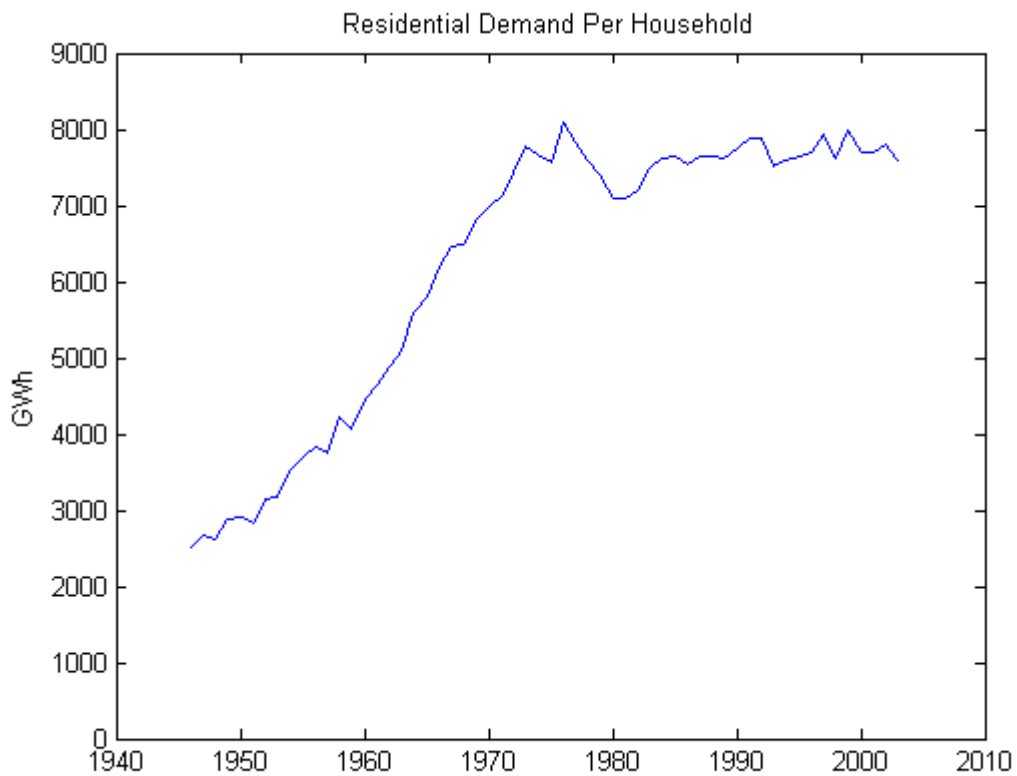
Generally it is desirable to include as wide a span of data as possible in a model to get the best estimate of the relationship between the drivers and the variable being modelled. However, in some cases it is better to truncate the range of data being used where there have been step changes in the underlying relationships due to factors that can not be easily modelled. Historical demand data is available in a consolidated form from the 1920s. A simple observation of historical residential demand shows a clear step change in the rate of growth experienced before and after the mid 1970's. The following diagram shows residential demand by year.



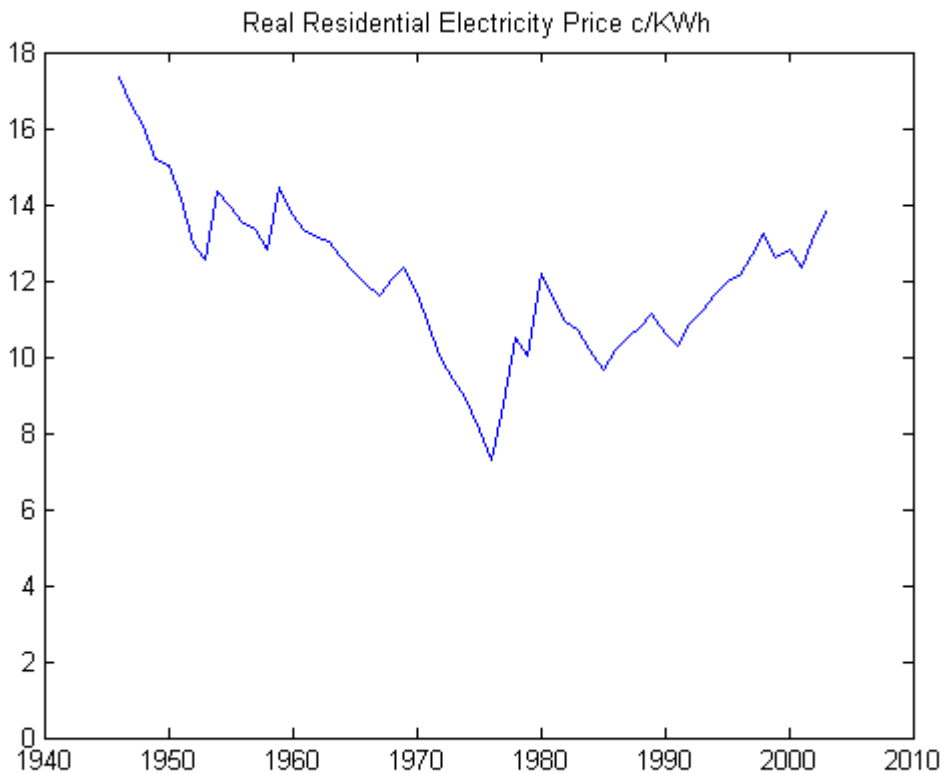
In the mid 1970's there was a shift away from an environment of exponential growth to a more linear growth rate. This is further illustrated when the relationship between domestic demand and some of the key drivers is examined. The following chart shows the change in relationship between domestic electricity demand and Real GDP (GDP is shown on the X axis and Domestic Demand on the Y axis).



At lower levels of GDP the ratio between GDP and residential demand is quite different in nature than the ratio at higher levels of GDP. Examining a plot of Residential Demand per Household highlights the extent of the underlying change.

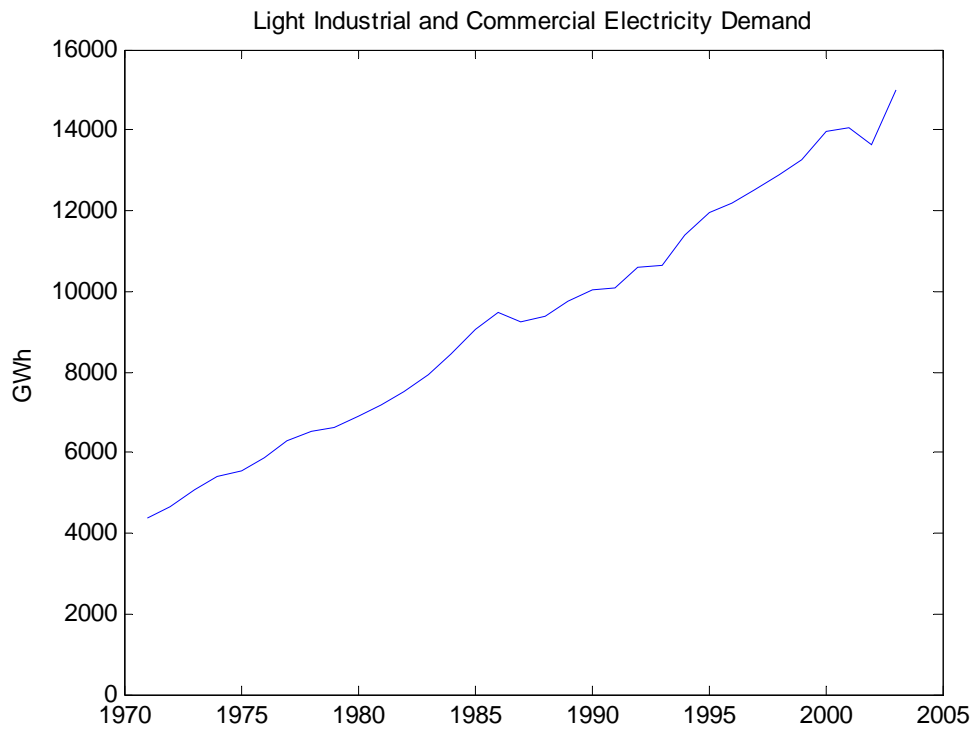


Demand per household has risen rapidly as households have gone from a state of having few electricity using appliances in the post war period, to the point of reaching a state of “saturation” in the 1970s. This has been bolstered by the large reduction in real electricity prices that occurred across that period as shown below.

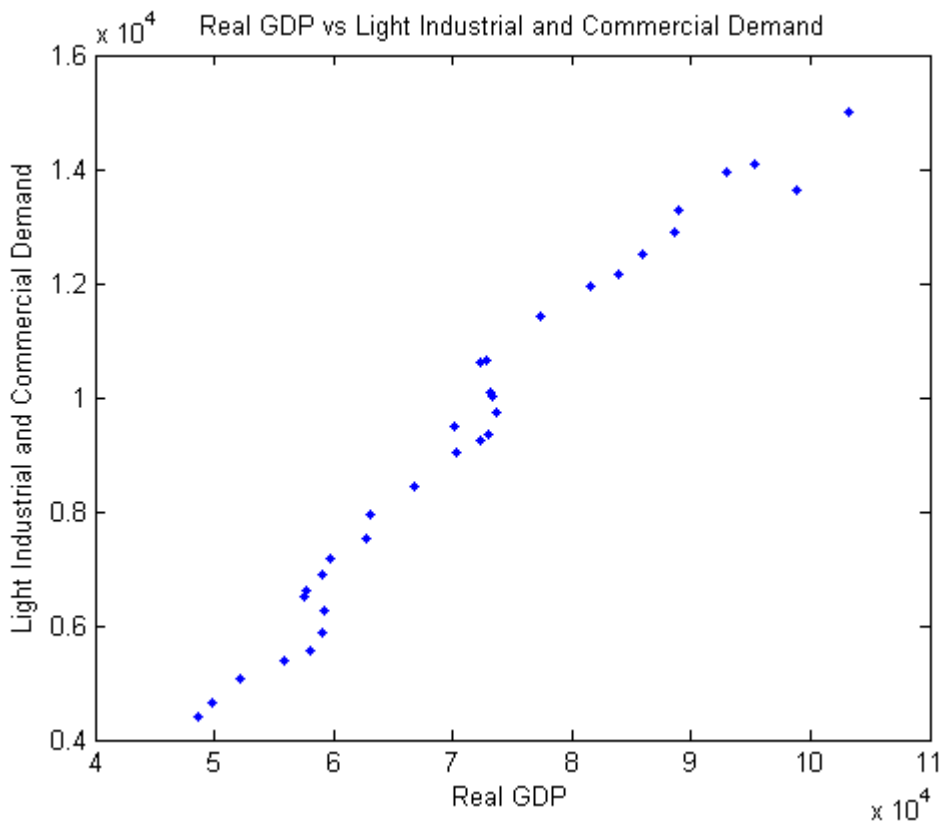


It is possible to construct models that take into account the saturation effect illustrated above (such as using a logistic model). Given the desire to keep the modelling approach as simple as possible we have chosen to base the residential modelling on data since the mid seventies rather than incorporating data prior to that date and using a more complex approach.

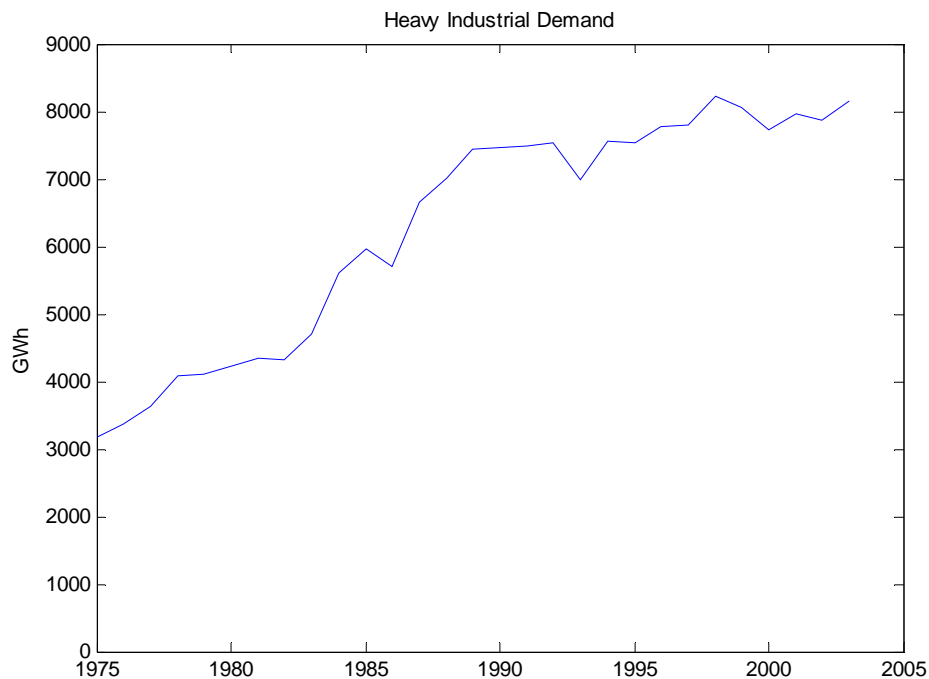
Consistent light industrial and commercial demand data is available from 1971 and is shown in the following graph.



There are no obvious step changes in this case. The peak in 1986 appears to be a categorisation issue between heavy and light industrial. A simple plot of Real GDP against demand (below) shows no obvious change in relationship. The full data set from 1971 has been used for modelling light industrial and commercial demand.



Heavy industrial demand covers the large industrial direct connect customers. The following graph shows heavy industrial demand since 1975.



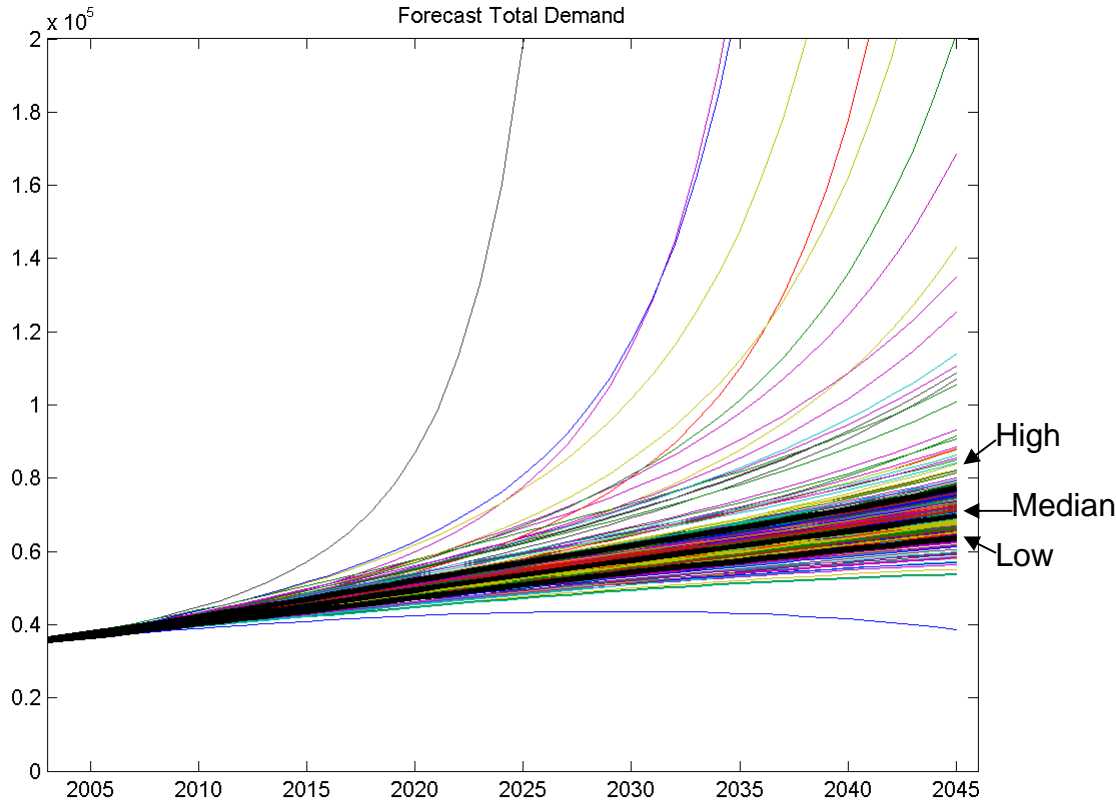
In the case of heavy industrial demand, there is a significant change in growth rates over the period covered by the historical data, with the break point sitting in the late 1980s. The relatively rapid growth seen up until then reflects the impact of government policies at the time, where indirect subsidies and direct support for “Think Big” projects resulted in the building and/or expansion of a number of heavy industrial users such as the aluminium smelter, the Glenbrook steel mill, and wood and paper processing mills. Growth in the heavy industrial sector slowed significantly in the late 1980s, although it continues to show a gradual upwards trend. Modelling of heavy demand has been based on data from 1989 onwards.

Modelling uncertainty

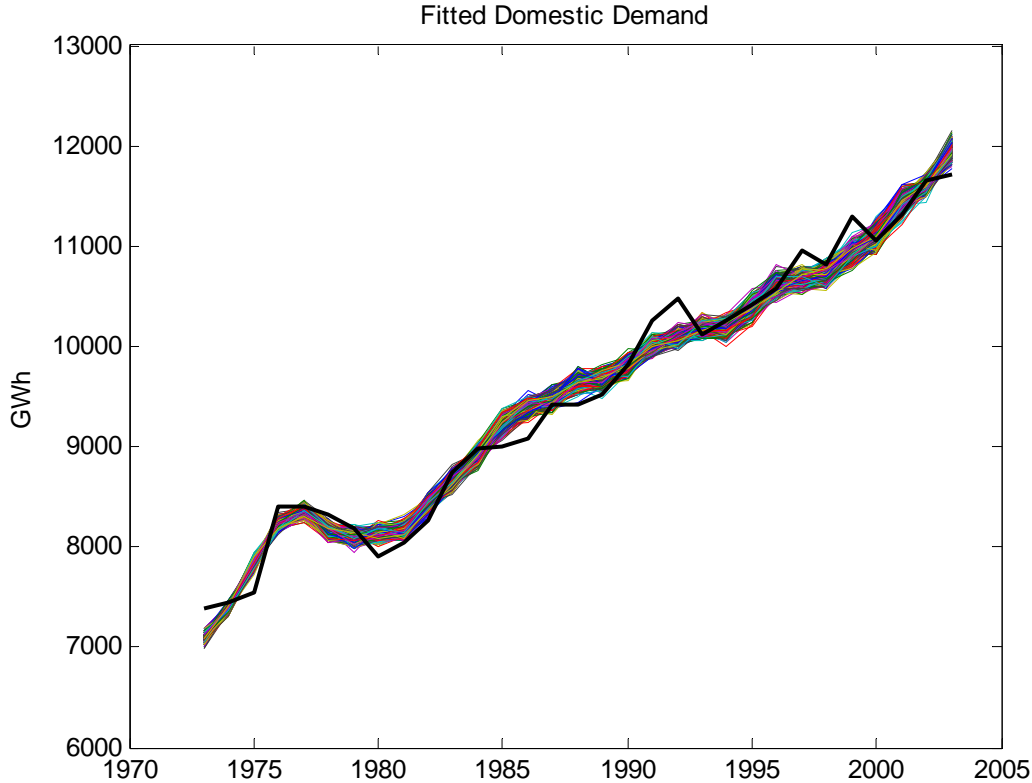
The goal of the model assessment was to select a model that fitted the historical data well while minimising forecast uncertainty. As noted earlier, model accuracy needs to be balanced against the requirement that the model be intuitive and easy to explain.

In order to use a consistent approach across the various alternative model types, modelling error was assessed using Monte-Carlo analysis of the input series. This technique involves estimating distributions for the underlying explanatory variables (i.e. the drivers) used in the model. The model is then re-run many times, substituting the actual input data with data randomly drawn from the estimated distributions. This provides a range of forecasts that confidence limits can be based on. This technique also provides multiple series of modelled data that can be compared for fit against the actual results. These can be used to assess the stability of the model to small changes in the underlying inputs.

The following graph illustrates the forecast results from one of the models together with 90% confidence limits for the forecasts based on modelling error.



The following graph illustrates the fitted domestic demand for the same model used in the forecast graph above.



Estimating Input Distributions

Two alternative Monte Carlo techniques were used for assessing model error. Initially a “bootstrap” version of the model was developed. This technique involves re-estimating the model but using an artificial set of inputs drawn randomly from the original set of inputs¹. This approach proved to be highly unstable because of the small number of data points available for modelling. The second approach was to create synthetic distributions for each of the input series². Because of the nature of the inputs, the approach used was to examine the variation in the individual inputs compared to the short term trends in the input (the variation between the input in any given year and a 5 year moving average was used). The variation for each year was used to create an artificial distribution for the input. The original inputs were then adjusted to incorporate the artificial variation and the model re-estimated.

Model selection

A number of model structures were considered as part of the review. Each model used a range of input variables selected or derived from the core drivers and could be split into four main categories:

Single-stage regression with linear inputs/outputs	Two-stage regression with linear inputs/outputs
Single-stage regression with log inputs/outputs	Two-stage regression with log inputs/outputs

The single stage regression is where a single regression is carried out to determine the relationship between the various inputs and the modelled output. Two stage regression models are used (in this context) in order to use a lagged variable in the modelling. Demand in any given year is subject to variation caused by factors that are impractical to model. The first stage of the two stage approach “smoothes” the demand so that it can be used as a lagged variable in the regression carried out in the second stage.

Log models convert all the inputs and outputs to their log values before carrying out the regression analysis, and then convert the resulting log values back. Log models are often a good choice when growth in the inputs and outputs is compounding in nature (such as is the case with GDP and population). Linear models use the raw input series rather than converting them to logs first.

¹ If the original data set contained 20 groups of points for example (such as GDP, population and demand for a given year), then each synthetic set would consist of 20 groups randomly selected **with replacement** from that original set. See “*Numerical recipes in C : the art of scientific computing / William H. Press: : [et al.]. – 2nd ed.* Chapter 15.6, available at http://physics.dit.ie/resources/physicstech4/comp/num_recipes/

² See Chapter 15.6 Numerical recipes in C (see reference 1 above)

The models were initially assessed on a number of criteria in order to narrow them down including:

- fit to historical data
- stability given uncertainty in the inputs
- the performance of the model when the input data is truncated (say to 5 years ago) and the model re-estimated and the forecasts compared to the actual values, and
- the t-values of the individual coefficients (which measure how significant the individual inputs are to the forecast values)

Modelling for the three sectors was considered separately – i.e. there was no deliberate attempt to arrive at common model structures for the different sectors.

Residential Model

As noted earlier, the range of models that could be explored was limited to a large extent by the long term historical and forecast figures available from reliable sources. These included historical residential demand, GDP, population, number of domestic residences, prices, CPI, and temperature data.

All of the residential models initially adjust raw demand to account for some of the impact of the average monthly temperatures for the year (generally this adjustment is relatively small). The adjustments are based on the temperature component identified using a Kalman filter on historical data³.

Most of the models include a shortage variable which essential removes from the regression results those years that “shortages” have occurred in. This ensures that demand is not biased downwards as a result of extra-ordinary circumstances that we do not wish to incorporate into normal planning.

The following residential models were explored as part of the review. Some of these were eliminated fairly quickly and thus have not been analysed to any great depth. A summary of the more successful residential models is at the end of this section.

Single Stage Linear Models

- V 1. Dependant (Forecast) Variable: Total Residential Demand
 Explanatory Variables: Constant, Domestic Residences, Total Energy Cost, Shortage

This model uses aggregate totals as input variables in order to model total residential demand. The t-statistic for total energy cost was not very good, and the truncated version of the model did not perform especially well when compared to the actual demand. The model did not fit the historical data well compared to the better models.

³ Kalman filters are used to break down time series into a number of components (in this case, a base trend, seasonal, weekday, daylight saving, temperature and a random component) which can then be reconstructed in order to produce synthetic or forecast series. The initial Kalman analysis used for the temperature adjustment was carried out by Jonathan Lermitt in 2002/2003 on behalf of EECA

- V 2. Dependant (Forecast) Variable: Demand Per Capita
Explanatory Variables: Constant, GDP/Capita, Households/Capita, Real Price, Shortage

This model estimates Demand Per Capita (which is then multiplied back up by historical/forecast population to reach national level numbers). Household Per Capita is used rather than the previously used People Per Household in order to retain consistency within the model (although the end effect is simply to change the coefficient value rather than the actual results). The model yielded good t-statistics for the key inputs, fitted the historical data well, and the truncated version of the model performed well. This model is compared further in the summary section below.

- V 3. Dependant (Forecast) Variable: Total Residential Demand
Explanatory Variables: Constant, GDP, Domestic Residences, Total Energy Cost, Shortage

This model uses inputs that are highly correlated (GDP and Domestic Residences). The fit to historical data and the t-statistics are reasonable however the truncated version of the model performed badly.

- V 4. Dependant (Forecast) Variable: Total Residential Demand
Explanatory Variables: Constant, GDP, Total Population, Total Energy Cost

This model does not fit the historical data well compared to the better models and suffers from high correlation between GDP and population (reflected in the relatively low t-statistic for each).

Single Stage Log Models

- V 1. Dependant (Forecast) Variable: Total Residential Demand
Explanatory Variables: Constant, Domestic Residences, Real Price, Shortage

This model is similar to the single stage linear V1 model although Real Price has been substituted for Total Energy Cost (using a log model means that the results are the same and the use of real price simplifies the modelling of the forecasts). This model performed well, with good t-statistics, a good fit on historical data and a good fit for the truncated forecast. The model is compared further below.

- V 2. Dependant (Forecast) Variable: Demand Per Capita
Explanatory Variables: Constant, GDP/Capita, Households/Capita, Real Price, Shortage

This is a log version of the V2 linear model above. This model also yielded good results and is compared in the summary section below.

- V 3. Dependant (Forecast) Variable: Total Residential Demand
Explanatory Variables: Constant, GDP, Domestic Residences, Real Price, Shortage

This is a log version of the linear V3 model and like the log V1 model substitutes Real Price for Total Energy Cost. The truncated version of the model performed well. The fit to historical data is reasonable although the t-statistic for GDP was not good (GDP and Domestic Residences are highly correlated).

- V 4. Dependant (Forecast) Variable: Total Residential Demand
Explanatory Variables: Constant, GDP, Total Population, Total Energy Cost

The log version of the linear V4 model does not perform well. The historical fit is not good compared to the better models, the truncated forecast is significantly higher than the actual demand, and the coefficients have poor t-statistics.

Two Stage Linear Models

- V 1. Dependant (Forecast) Variable: Demand Per Capita
Explanatory Variables: Constant, GDP/Capita, Households/Capita, Real Price, Shortage and Lagged Demand Per Capita

This model fitted the historical data well but like many of the two stage models the model proved to be unstable when variation was introduced into the inputs, with the result that the forecasts produced varied widely. The truncated forecast was reasonable, although the demand spread could be seen even over the 5 year period. The t-statistics for most of the coefficients were poor.

- V 2. Dependant (Forecast) Variable: Total Residential Demand
Explanatory Variables: Constant, GDP, Households, Total Energy Cost, Shortage and Lagged Demand

Again, this model fitted the historical data reasonably well but the forecasts were unstable. The truncated forecast was poor as were the t-statistics for the coefficients. The lag coefficient was over 1.

- V 3. Dependant (Forecast) Variable: Demand Per Capita
Explanatory Variables: Constant, Households/Capita, Real Price, Shortage and Lagged Demand Per Capita

In this case the forecasts were relatively stable however the model had a poor fit to the historical data and the lag coefficient was surprisingly low at 0.14. The truncated forecast was not very good and the t-statistics for the coefficients were low.

- V 4. Dependant (Forecast) Variable: Total Residential Demand
Explanatory Variables: Constant, Households, Total Energy Cost, Shortage and Lagged Demand

The fit to historical data for this model was average compared to the better models. In some cases the forecasts oscillated wildly in response to variation introduced into the inputs. The truncated forecast was significantly higher than

the actual demand. The coefficient for the lag is actually negative (-0.25) and the t-statistics for the coefficients were low.

Two Stage Log Models

- V 1. Dependant (Forecast) Variable: Demand Per Capita
Explanatory Variables: Constant, GDP/Capita, Households/Capita, Real Price, Shortage and Lagged Demand Per Capita

This is the existing Transpower residential model and is essentially a log version of the Linear Two Stage V1 model above. This model fitted the historical data well but also proved to be unstable when variation was introduced into the inputs. The truncated forecast was reasonable, although the forecast spread was similar to the linear version of the model. The t-statistics for most of the coefficients were low as the forecasts are dominated by the lagged demand variable. This model is included in the summary section below as a benchmark for the alternative models.

- V 2. Dependant (Forecast) Variable: Total Residential Demand
Explanatory Variables: Constant, GDP, Households, Real Price, Shortage and Lagged Demand

This is a log version of the Linear Two Stage V2 model with Real Price being substituted for Total Energy Cost. This model also fitted the historical data well but with unstable forecasts. The truncated forecast was fairly good although the t-statistics for the coefficients were average. The lag coefficient was 0.41.

- V 3. Dependant (Forecast) Variable: Demand Per Capita
Explanatory Variables: Constant, Households/Capita, Real Price, Shortage and Lagged Demand Per Capita

A log version of the V3 Two Stage model above, the model had a relatively poor fit to the historic data. In this case the forecasts were stable, although the truncated demand forecast is somewhat higher than the actual demand. The lag coefficient was low at 0.18 and the t-statistics for most of the coefficients were not good.

- V 4. Dependant (Forecast) Variable: Total Residential Demand
Explanatory Variables: Constant, Households, Real Price, Shortage and Lagged Demand

The fit to historical data for this model was good and the forecasts stable with the truncated forecast model giving a reasonable result. The lag coefficient though was very low (0.02) with a very low t-statistic.

Residential Model Summary

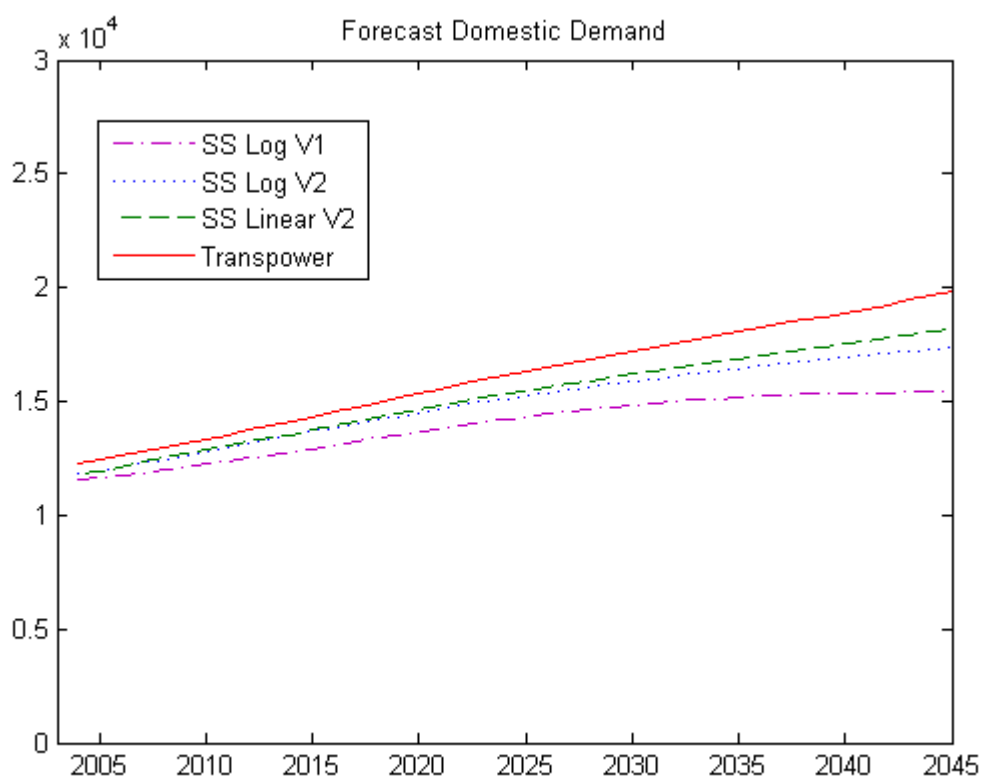
Of the models tested, three were considered to show reasonable promise with respect to their performance as assessed by the measures we used. The following table summarises the models and key statistics for each together with the existing

Transpower model. A graphical comparison of the various models including the Monte-Carlo runs can be found in Appendix B.

Model	2 Stage Log V1 (Transpower)				Single Stage Linear V2			
	Variable	coeff.	s.d.	t-stat	Variable	coeff.	s.d.	t-stat
Coefficients, standard deviations, and t-statistics	Constant	0.068	0.57	0.12	Constant	-0.057	0.23	-0.25
	log(GDP/Capita)	0.157	0.09	1.75	GDP/Capita	0.040	0.01	4.15
	log(HH/Capita)	0.104	0.32	0.33	HH/Capita	6.717	1.01	6.65
	log(Price)	-0.072	0.05	-1.36	Price	-0.043	0.01	-3.63
	Shortage	-0.007	0.01	-0.56	Shortage	-0.002	0.04	-0.05
	Lag	0.743	0.28	2.64				
R ²	0.9816				0.9790			
Adjusted R ²	0.9779				0.9758			
Durbin-Watson	1.6125 (d _L = 1.10916 d _U =1.81867)				1.3213 (d _L = 1.17688 d _U =1.73226)			

Model	Single Stage Log V1				Single Stage Log V2			
	Variable	coeff.	s.d.	t-stat	Variable	coeff.	s.d.	t-stat
Coefficients, standard deviations, and t-statistics	Constant	-7.659	0.62	-12.32	Constant	1.328	0.34	3.89
	log(HH)	1.125	0.05	24.32	log(GDP/Capita)	0.310	0.07	4.22
	log(Price)	-0.215	0.05	-4.42	log(HH/Capita)	0.864	0.14	6.31
	Shortage	0.001	0.14	0.09	log(Price)	-0.158	0.04	-3.70
					Shortage	-0.000	0.01	-0.01
R ²	0.9794				0.9793			
Adjusted R ²	0.9771				0.9761			
Durbin-Watson	1.1509 (d _L = 1.24371 d _U =1.65046)				1.3354 (d _L = 1.17688 d _U =1.73226)			

The following graph shows the forecast for each model.



As noted above, while the Transpower model fits the historical data reasonably well (as measured by the R^2 statistic⁴), the t-statistics⁵ for most of the variables in the model are low as they are swamped by the lagged variable. The model forecasts also proved to be very unstable when variation was introduced into the inputs (graphs showing the variation can be found in Appendix B).

There is little to choose between the other models purely based on the test statistics of each. The Single Stage Log V1 model shows autocorrelation⁶ between the errors (as measured by the Durbin Watson test). The Durbin Watson test results for the other models were inconclusive. The main concern with the Single Stage Log V1 model, aside from the Durbin Watson result, is that it ignores income effects on demand. Research carried out by BRANZ suggests that home heating temperatures during winter are below what would be expected in a cool climate. There is evidence of a trend towards an increase in average home temperatures in developed countries over time. While this is likely to be driven to some extent by gradual improvements in the housing stock, this is also believed to be attributable to increasing real wealth. We therefore believe that income driven demand is an important consideration for potential future growth in New Zealand.

The choice between the use of the log version or the linear version of the Single Stage V2 model is not a clear one. Both models yield much the same test results, with the log version of the model performing slightly better in general. Examination of the stability of the forecasts to input variation (see Appendix B) shows that the log version of the model performs marginally better than the linear version.

In terms of balancing the complexity of the model against its performance we believe the use of the slightly more complex log model is justified on the basis of its better overall results. Although the coefficient for the shortage variable in this model is not significantly different to zero, we have left the coefficient in for Monte-Carlo modelling purposes.

Commercial and Light Industrial Model

The range of models explored for the light industrial and commercial modelling is similar to those covered in the residential modelling, although the drivers used are generally limited to GDP, price and a shortage flag.

The following models were explored as part of the review.

⁴ The R^2 statistic measures the fit of the modelled data to the actual data. The statistic ranges between 0 and 1, with 1 representing a perfect fit. The Adjusted R^2 statistic is an extension of the R^2 statistic which corrects the result based on the number of explanatory variables being used.

⁵ The t-statistics measure the significance of the individual coefficients in the model. The t-statistic can be used in conjunction with the t-distribution to test to a specified level of confidence whether or not the coefficient is in fact different to 0. In general, the higher the t-statistic is, the more significant the coefficient. In most of the models assessed here the shortage variable has a poor t-statistic as there are only two shortage years in the source data, thus the t-statistic for this particular co-efficient has largely been ignored.

⁶ In this context, autocorrelation describes how closely related the error in one year is compared to the error in the previous year. High autocorrelation indicates that there are patterns in the errors that improved modelling may be able to capture. Values for the Durbin Watson test range between 0 and 4, where 0 indicates perfect correlation between the errors, 2 indicates no relationship, and 4 indicates a perfect negative relationship. If the Durbin Watson value is less than d_L then the test shows that significant positive autocorrelation exists, if it is greater than d_U then significant positive autocorrelation does not exist, and if it lies between d_L and d_U then the test is inconclusive.

Single Stage Linear Models

- V 1. Dependant (Forecast) Variable: Total Commercial and Light Industrial Demand
Explanatory Variables: Constant, GDP, Shortage

This model uses aggregate GDP as an input variable in order to model total commercial and light industrial demand. The model did not fit the historical data well as it responds to year on year movements in GDP that are not reflected in the demand figures. The t-statistic for GDP is good, although the truncated version of the model does not perform well when compared to the actual demand.

- V 2. Dependant (Forecast) Variable: Commercial and Light Industrial Demand Per Capita
Explanatory Variables: Constant, GDP/Capita, Shortage

This model uses GDP per capita as an input variable in order to model commercial and light industrial demand per capita. The t-statistic for GDP/Capita is good but, similar to the V1 model, the model did not fit the historical data well and the truncated version of the model produced forecasts higher than actual demand in the later years.

- V 3. Dependant (Forecast) Variable: Commercial and Light Industrial Demand Per Capita
Explanatory Variables: Constant, GDP/Capita, Real Price, Shortage

The t-statistic for GDP/Capita is good but the price t-statistic was very poor indicating little price responsiveness evident in the data. The results are therefore very similar to those in V2 above.

- V 4. Dependant (Forecast) Variable: Total Commercial and Light Industrial Demand
Explanatory Variables: Constant, GDP, Total Energy Cost, Shortage

This model fitted the historical data slightly better than the other models in the linear single stage group. The coefficient for Total Energy Cost is positive (not unexpected given the lack of price responsiveness demonstrated in V3 above). The t-statistics for the model are good, although the truncated version of the model, like the others in the group, overshoots actual demand in later years.

Single Stage Log Models

- V 1. Dependant (Forecast) Variable: Total Commercial and Light Industrial Demand
Explanatory Variables: Constant, GDP, Shortage

This is a log version of the Single Stage Linear V1 model above. The model suffers from similar problems with respect to fitting the historical data. The t-statistics for the model are good but the truncated forecasts significantly overshoot the actual demand in later years.

- V 2. Dependant (Forecast) Variable: Commercial and Light Industrial Demand Per Capita

Explanatory Variables: Constant, GDP/Capita, Shortage

This is a log version of the Linear V2 model above. The model fits the historical data poorly. Like most of the previous models the t-statistics are reasonable but the truncated forecasts significantly overshoot the actual demand. The forecasts show signs of exponential growth in later years.

- V 3. Dependant (Forecast) Variable: Commercial and Light Industrial Demand Per Capita
Explanatory Variables: Constant, GDP/Capita, Shortage

This is a log version of the Linear V3 model above. Again, the model fits the historical data poorly. The price coefficient t-statistic is low (and the coefficient is positive suggesting demand increases as the price goes up). The truncated forecasts overshoot the actual demand in later years.

- V 4. Dependant (Forecast) Variable: Total Commercial and Light Industrial Demand
Explanatory Variables: Constant, GDP, Real Price, Shortage

This is a log version of the Linear V4 model above but with Price substituted for Total Energy Cost. The model does not fit the historical data very well. The price coefficient is positive although the t-statistics for the model are fairly good. The truncated version of the model overshoots actual demand in later years.

Two Stage Linear Models

- V 1. Dependant (Forecast) Variable: Commercial and Light Industrial Demand Per Capita
Explanatory Variables: Constant, Year (1st stage only), GDP/Capita, Shortage and Lagged Demand Per Capita

This model, like the other commercial and light industrial two stage models uses Year as an explanatory variable in the first stage for smoothing purposes. This model fits the historical data very well. The t-statistic for GDP/Capita is not very good. The truncated model forecasts are better than those produced by the single stage models. The coefficient for lagged demand is 0.91.

- V 2. Dependant (Forecast) Variable: Total Commercial and Light Industrial Demand
Explanatory Variables: Constant, Year (1st stage only), GDP, Total Energy Cost, Shortage and Lagged Demand

This model fits the historical data well. Total Energy Cost has a poor t-statistic however the truncated forecast fits well compared to the other models. The coefficient for lagged demand is 0.77.

- V 3. Dependant (Forecast) Variable: Commercial and Light Industrial Demand Per Capita
Explanatory Variables: Constant, Year (1st stage only), GDP/Capita, and Lagged Demand

This is the existing Transpower model and is similar to the Two Stage Log V1 model above without the use of the shortage flag. The model fits the historic data well, although it does not respond to the 2001 shortage as would be expected. The t-statistic for GDP/Capita is poor. The truncated model forecast

is similar to those produced by other two stage models but the forecast is less stable producing some unusual results in some cases. The coefficient for lagged demand is 0.96.

- V 4. Dependant (Forecast) Variable: Commercial and Light Industrial Demand Per Capita
Explanatory Variables: Constant, Year (1st stage only), GDP/Capita, Real Price, Shortage and Lagged Demand

The model fits that historic data well, however the t-statistic for GDP/Capita is low and for Price very low (with the coefficient being positive). The truncated model forecast was good. The coefficient for lagged demand is 0.92.

- V 5. Dependant (Forecast) Variable: Total Commercial and Light Industrial Demand
Explanatory Variables: Constant, Year (1st stage only), GDP, Shortage and Lagged Demand

This is a model based on V2 above but excluding the Total Energy Cost variable due to its poor t-value. This model fits the historical data well and has good t-statistics. The truncated forecast fits well compared to the other models. The coefficient for lagged demand is 0.84.

Two Stage Log Models

- V 1. Dependant (Forecast) Variable: Commercial and Light Industrial Demand Per Capita
Explanatory Variables: Constant, Year (1st stage only), GDP/Capita, Shortage and Lagged Demand Per Capita

This is a log version of the Two Stage Linear V1 model. The model does not fit the historical data as well as the linear model, showing a tendency towards exponential growth that is reflected in the forecasts. The truncated model forecasts significantly overshoot the actual demand and the t-statistic for GDP/Capita is very poor. The coefficient for lagged demand is 0.92.

- V 2. Dependant (Forecast) Variable: Total Commercial and Light Industrial Demand
Explanatory Variables: Constant, Year (1st stage only), GDP, Real Price, Shortage and Lagged Demand

This model fits the historical data well. The t-statistics for the coefficients are generally poor, with the exception of lagged demand (which has a coefficient of 0.87). The truncated forecasts are high compared to actual demand and the forward forecasts are highly exponential.

- V 3. Dependant (Forecast) Variable: Commercial and Light Industrial Demand Per Capita
Explanatory Variables: Constant, Year (1st stage only), GDP/Capita, and Lagged Demand

Like the V1 model above this model shows a tendency towards exponential growth and does not fit the historical data particularly well. The truncated model forecasts are not good and the t-statistic for GDP/Capita is poor. The co-efficient for lagged demand is 0.95. The forward forecasts are unstable.

- V 4. Dependant (Forecast) Variable: Commercial and Light Industrial Demand Per Capita

Explanatory Variables: Constant, Year (1st stage only), GDP/Capita, Real Price and Lagged Demand

The model fits historic data well, and the truncated forecasts are reasonable, although they exhibit more spread than the other models. The forecasts show significant exponential growth and the t-statistics for GDP/Capita and Price are both poor. The co-efficient for lagged demand is 0.92.

Commercial and Light Industrial Model Summary

None of the models performed exceptionally well. The single stage models typically over-responded to GDP as an input and consequently produced poor short term forecasts. The accuracy of the models with respect to short to medium term forecasts is an issue for some applications (such as reserves management).

The relatively stable nature of commercial and industrial demand means that the lagged demand variable tends to swamp the other explanatory variables in the two stage models (as borne out by the t-statistics for those models).

The following tables summarise the results from the best performing models of those assessed.

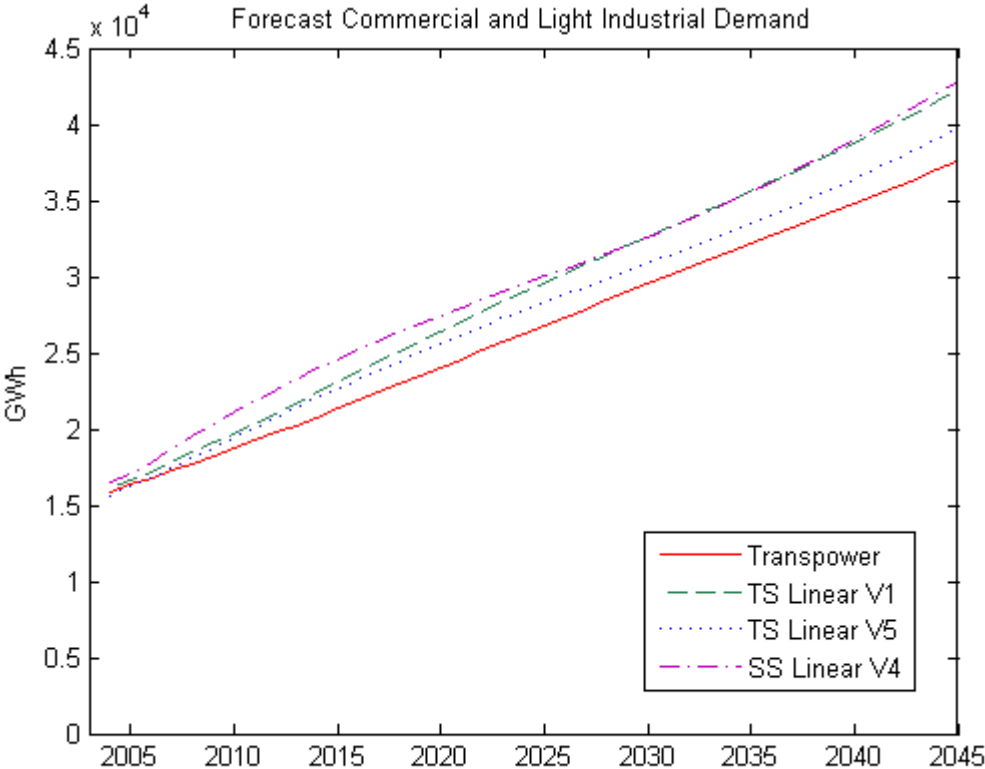
Model	2 Stage Linear V3 (Transpower)				2 Stage Linear V1			
	Variable	coeff.	s.d.	t-stat	Variable	coeff.	s.d.	t-stat
Coefficients, standard deviations, and t-statistics	Constant	-0.048	0.32	-0.15	Constant	-0.228	0.28	-0.81
	GDP/Capita	0.011	0.03	0.41	GDP/Capita	0.026	0.23	1.14
	Lag	0.960	0.09	10.22	Shortage	-0.140	0.05	-2.70
					Lag	0.912	0.08	11.10
R ²	0.9896				0.9915			
Adjusted R ²	0.9888				0.9906			
Durbin-Watson	0.8786 (d _l = 1.30932 d _u =1.57358)				0.9000 (d _l = 1.24371 d _u =1.65046)			

Model	2 Stage Linear V5				Single Stage Linear V4			
	Variable	coeff.	s.d.	t-stat	Variable	coeff.	s.d.	t-stat
Coefficients, standard deviations, and t-statistics	Constant	-809.579	492.09	-1.65	Constant	-5930.000	413.98	-14.32
	GDP	0.036	0.02	2.23	GDP	0.179	0.01	19.71
	Shortage	-333.382	155.76	-2.14	Energy Cost	2.020	0.49	4.08
	Lag	0.843	0.08	10.79	Shortage	-104.980	285.61	-0.37
R ²	0.9934				0.9802			
Adjusted R ²	0.9927				0.9781			
Durbin-Watson	1.2652 (d _l = 1.24371 d _u =1.65046)				0.5473 (d _l = 1.25756 d _u =1.65110)			

A number of the models assessed included a price or total energy cost variable. In most cases the price variable proved to be insignificant, and in some cases the coefficient was the wrong sign (i.e. suggesting that as electricity prices increased,

demand increased). Although it would be useful to incorporate price effects in the forecast, the historical data does not provide sufficient information to do so. It should be recognised though that for most industries in this sector, electricity costs are only a small proportion of total operating costs and the ability to switch fuel types is limited at best. Electricity prices on the whole are therefore not expected to have a significant effect on demand in this sector, although they may result in some longer term impacts associated with efficiency savings and fuel switching where it is feasible.

The following table shows the forecasts for each of the models.



The Transpower model gives the lowest of the forecasts. The only difference between the Transpower model and the Two Stage Linear V1 model is the introduction of a shortage flag to adjust for those years affected by supply “shortages”. The results suggest that Transpower’s forecasts are pulled down by the lower demand in those years. Examination of the various plots in Appendix C for the models illustrates the performance and fits of the short listed models. The Two Stage Linear V5 model gives the best t-statistics of the two staged models, best r-squared of all the models and the least autocorrelation in the errors between the fitted and actual demand (the Durbin-Watson test in this case is (just) inconclusive whereas the other models show significant positive autocorrelation). The forecast from this model is also the least sensitive to input variation. We believe that the Two Stage Linear V5 model is therefore the most appropriate of those assessed.

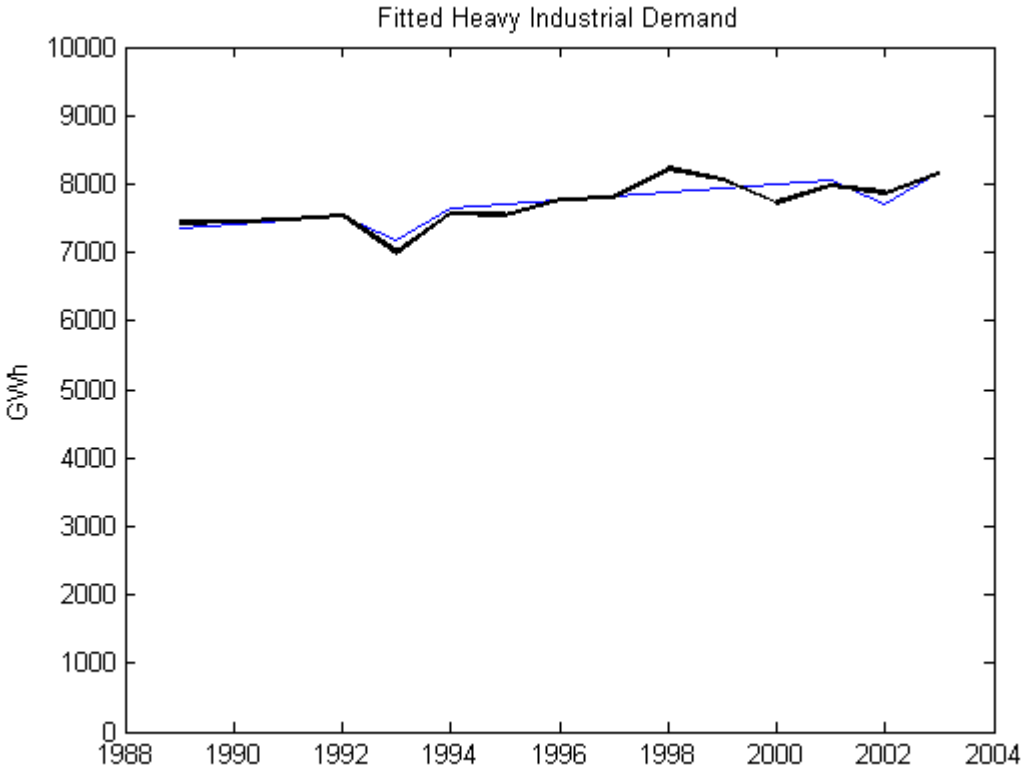
Heavy Industrial Model

Modelling heavy industrial use is more problematic than the other sectors as it is not easily relatable to underlying high level drivers such as GDP or population. Growth in

this sector is likely to be driven by conditions in the markets for the respective end products and by factor costs, such as labour, energy costs and the cost of raw materials.

Information on expectations of growth within this sector may be yielded through industry specific studies. At this stage though, there is insufficient information to allow anything but a simple extrapolation of recent growth trends to be used (the same approach is used in the Transpower demand forecast).

The following chart shows heavy industrial demand from 1989 and the fitted trend (incorporating a “shortage” flag to adjust for those years where there were supply issues).



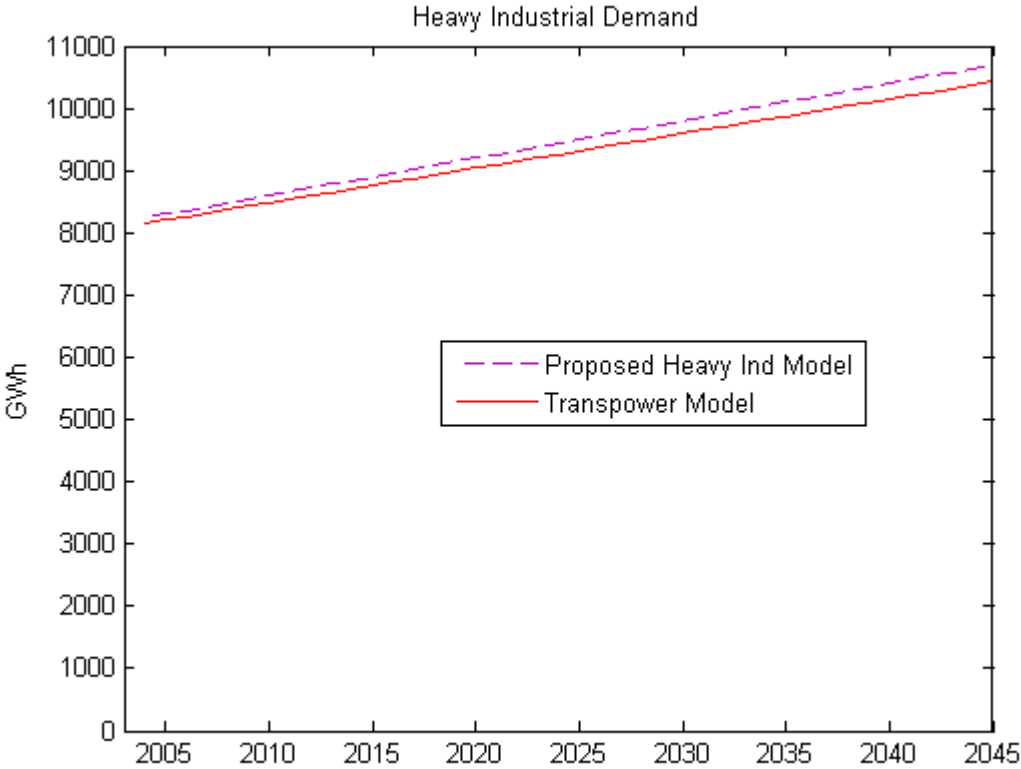
The statistics for the regression carried out to fit the above trend are listed in the following table:

Model	Single Stage Heavy Industrial			
	Variable	coeff.	s.d.	t-stat
Coefficients, std deviations, and t-statistics	Constant	-111930	19424	-5.76
	Year	59.964	9.7	6.16
	Shortage	-416.478	123.7	-3.37
R ²	0.7875			
Adjusted R ²	0.7489			
Durbin Watson	1.7755 (d _L = 0.94554 d _U = 1.54318)			

The Ministry of Economic Development’s 2003 Energy Outlook publication makes a number of forecasts of total energy use to 2025. Included in these are forecasts for

the Basic Metals and Forestry sectors. The forecasts yielded by the above model are largely consistent with the electricity component of the MED forecasts for those sectors.

The following graph shows the forecast for heavy industrial demand from the above model and compares it to the Transpower forecast. The key difference between the two models is that the Transpower forecast does not include a flag to adjust for shortage years.



Total Demand Forecast

Total forecast electricity demand at grid exit point consists of the total of the three sectors, plus an estimate of local lines losses, less demand that is met by embedded generation.

In summary, total demand can be described as

The residential model where:

$$\log(\text{Resid.Demand/Capita}) = 1.328 + 0.310 \times \log(\text{GDP/Capita}) + 0.864 \times \log(\text{Households/Capita}) + -0.158 \times \log(\text{Real Price}) + 0.000 \times \text{Shortage Flag}^7$$

and

$$\text{Total Residential Demand} = e^{\log(\text{Resid.Demand/Capita})} \times \text{Population}$$

The commercial and light industrial model where:

⁷ As noted earlier we have left this coefficient in the model for Monte-Carlo modelling purposes.

$$\text{Total Comm. and Light Ind. Demand} = -809.6 + 0.036 \times \text{GDP} + -333.4 \times \text{Shortage Flag} + 0.843 \times \text{Lagged Demand}$$

where Lagged Demand is from a smoothed series where :

$$\text{Smoothed Demand} = -463424.9 + 235.9 \times \text{Year} + 0.056 \times \text{GDP} + -330.76 \times \text{Shortage Flag}$$

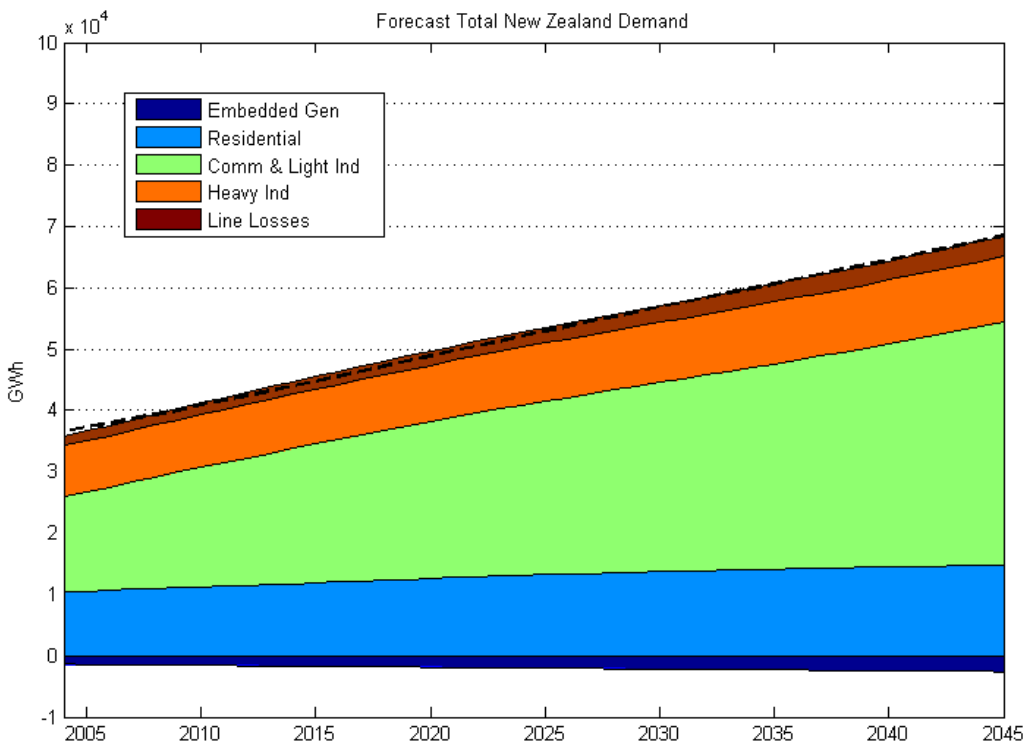
The heavy industrial model where:

$$\text{Total heavy industrial load} = -111930 + 59.964 \times \text{Year} + -416.478 \times \text{Shortage Flag}$$

Local lines losses are assumed to continue at their recent historical level of between 5 and 6% (5.75% is used – actual losses vary slightly between years). Actual future losses are subject to conflicting drivers. As lines asset utilisation increases, average losses would be expected to increase. However, improvements in the quality of local network assets are expected to at least offset this.

Growth in embedded generation is highly uncertain and will be driven by changes in technology costs for small scale generation compared to improvements in larger scale generation. For the purposes of the base demand forecast it has been assumed that embedded generation grows in proportion to total demand (i.e. it remains at around 4-5% of total generation).

The following graph shows the various components of the mean demand forecast.



The dotted black line shows the total forecast using the 2004 Transpower model.

Forecast Input Uncertainty

The electricity demand forecasts are heavily dependant on the forecasts of the inputs used in the models. To assess the implications of uncertainty in the input forecasts it is necessary to: a) establish a view as to the likely variation in the underlying inputs; and b) model demand incorporating the input variation to establish the resulting impact on the demand forecasts.

The Monte-Carlo models built to assess modelling uncertainty were extended to incorporate forecast input uncertainty.

Uncertainty in the various inputs has been assessed separately.

GDP : Because of the close relationship between total GDP, Population and Housing, uncertainty in GDP has been broken down into three components when assessing uncertainty – population, productivity, and a random component. The population component is kept consistent with variation introduced in the Population section below. Productivity variation is handled by scaling productivity for all years by a random factor drawn from a distribution based on an estimated historical range. The third, random, component is intended to provide some year on year variation resulting from external causes such as changes in the international environment. In this case historical GDP variation has been used to estimate an appropriate distribution which is then applied to each individual year.

Households : Like GDP there is a relationship between population and the number of households. Uncertainty in households has been broken into two components, population uncertainty which is kept consistent with Population below, and a household size component. Household size (people per household) is handled in a similar way to productivity variation in GDP above, where a factor drawn from an estimated distribution is used to scale the household size across all years in each individual Monte-Carlo run. In this case though, the change in household size is phased in over the course of the forecast.

Population : Population variation is handled by using scenarios prepared by Statistics New Zealand to estimate a distribution for population growth over the period being modelled, and then applying a factor drawn from the distribution across all years in each Monte-Carlo run. While it is outside the scope of the current modelling, establishing a more comprehensive population model and introducing randomness into birth-rates, death-rates and immigration rates may be explored in the future.

Price : Price uncertainty is modelled by estimating a distribution to apply to average price movements and then scaling each year by the same factor drawn from that distribution for each Monte-Carlo run.

Price elasticity : Price elasticity of demand is a measure of how responsive demand is to the price changes above. In the case of Log based models, the coefficient for the price variable provides an indication of the underlying price elasticity. The price coefficient in the residential mode is -0.16 which indicates that a 10% increase in price would result in a 1.6% decrease in residential demand in the short to medium term. Results from international studies vary widely. EPRI⁸ reported results ranging

⁸ EPRI (1989) *Residential End-Use Energy Consumption: A Survey of Conditional Demand Estimates*, Palo Alto, CA: Electric Power Research Institute, Report CU-6487 (cited in Reiss, P.C. and M.W. White (2002), *Household Electricity Demand, Revisited*, Stanford University)

from -0.15 to -0.35 for analyses carried out by electricity utilities. Because residential demand is around 35% of total demand at present, the impact on total demand of assuming different price elasticities is muted. With a price elasticity of -0.16, a 50% price increase reduces total demand by around 2.8%. A price elasticity of -0.35 would produce a reduction of 6.1% in total demand given the same 50% increase in price.

As noted earlier, light industrial and commercial consumers have been largely unresponsive to long term changes in price, mainly because in most cases electricity represents a small proportion of total operating costs and the ability to switch to alternative energy sources is limited. Changes in price are more of an issue for those large industrials where electricity is a significant contributor to total production costs. Major changes in demand resulting from the exit or entry of large industrials, or changes in residential demand elasticity resulting from significant social or technology changes will be handled through scenario analysis carried out as part of the wider transmission planning activities.

Shortages : While the shortage flag used in the modelling allows the introduction of shortage years into the forecasts, its primary use is to ensure that the historical shortage years do not bias the forecasts downwards. We have not included forecast variation associated with possible future shortage years⁹.

Energy Intensity : The historical data the models have been estimated from incorporate past energy efficiency improvements resulting from changes in policies and standards, technology changes, social changes and the like. The forecasts therefore reflect an ongoing underlying rate of efficiency improvement. Step changes in energy efficiency resulting from policy initiatives that are significantly different to past changes are not modelled explicitly as part these forecasts. Where a demonstrable change in future demand resulting from an expected policy change can be robustly established and independently confirmed, we will consider incorporating that change into the forecasts as an explicit adjustment. Demand changes will need to be clearly separated from those changes that would have occurred anyway in the absence of the policy, and the policy would need to be clearly different from previous policies rather than just an evolution of past changes. Incorporating “possible” changes into the forecast is inappropriate as doing so increases the risk of under estimating future demand, and therefore increases the risk of constructing an inadequate transmission system.

The possible impacts of broader technology and social changes that may impact on electricity demand through changes in energy intensity will be dealt with through scenario analysis.

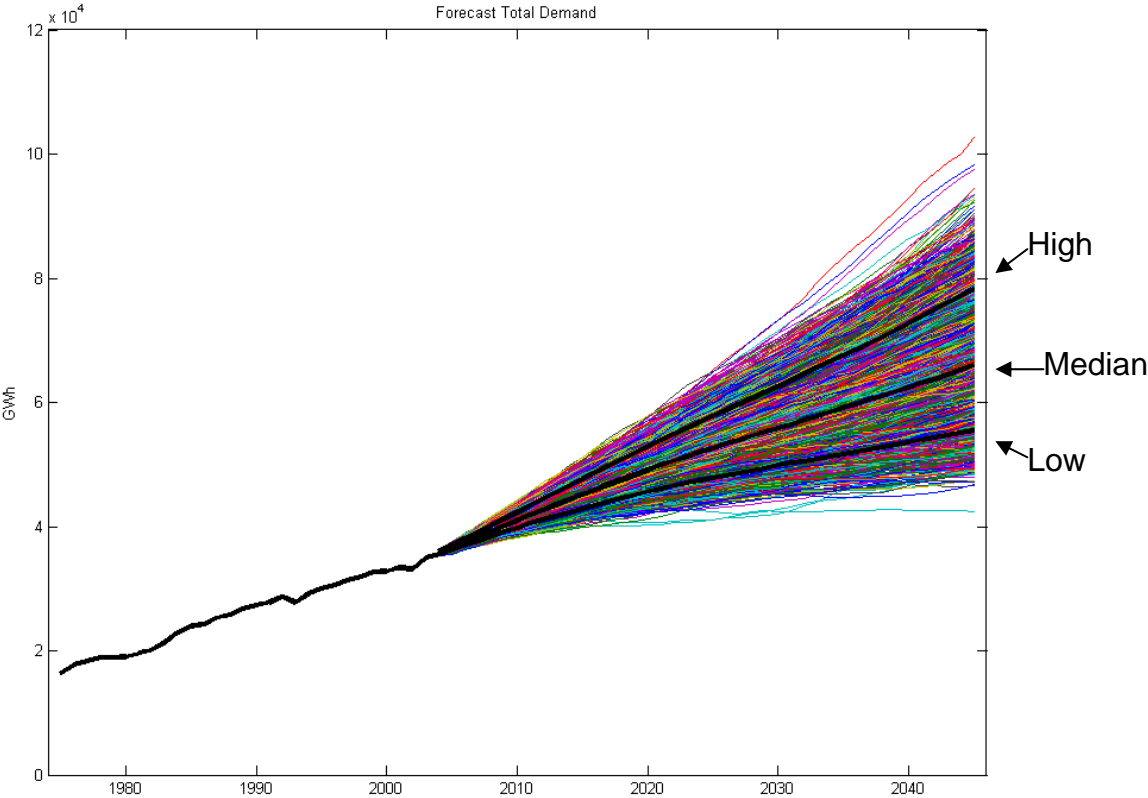
Embedded Generation : The balance between embedded and grid supplied generation will be largely determined by a mix of future technology and input cost changes. A good illustration of the tension between technology costs and other factors can be seen with wind generation. Up until a few years ago it was a widely held view that wind generation would typically be connected to local line networks. Economics of scale have resulted in many new wind farms (both overseas and recently in New Zealand) now being installed at a size where grid connection is more appropriate. We have not attempted to model the uncertainty in embedded

⁹ The approximate 1 in 60 year security criteria used in planning means that the impact of this is negligible.

generation explicitly in the demand forecasting model but will consider the impact of possible changes in embedded generation using scenario analysis.

Combined Forecast Uncertainty

Total New Zealand demand with 90% confidence limits for forecast uncertainty is shown in the following graph.



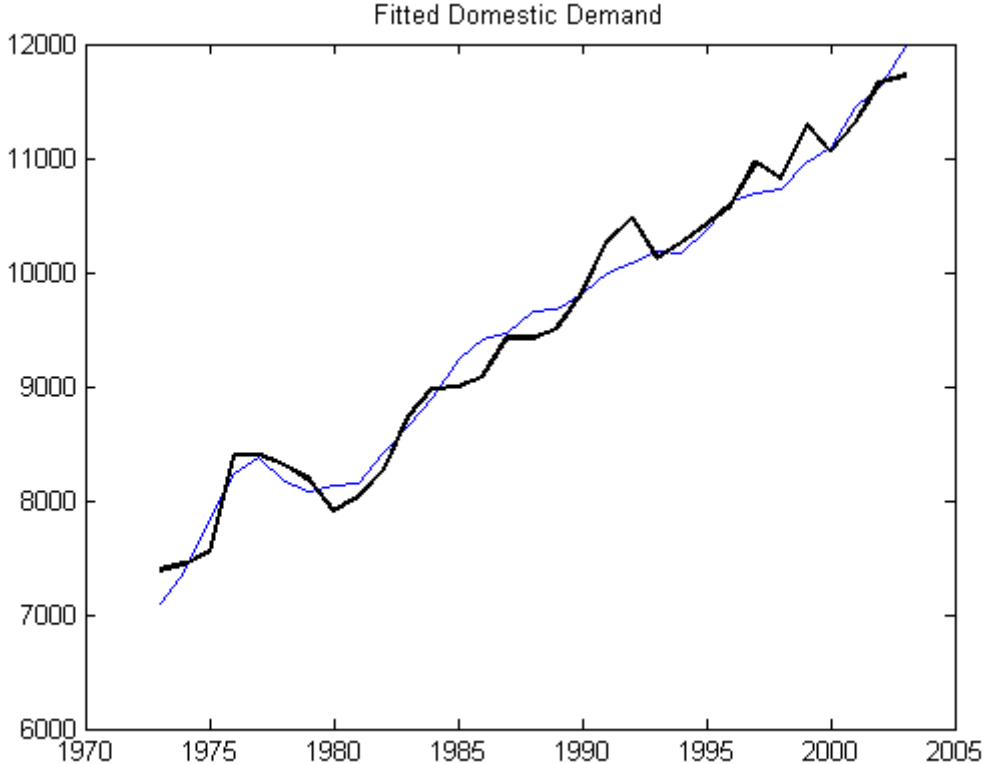
The graph demonstrates the uncertainty in demand associated with both model estimation and the forecast inputs. Uncertainty from sources such as step changes in energy efficiency trends, embedded generation patterns or changes in responsiveness to price movements are not included and will be dealt with using scenario analysis.

Appendix A: Key Drivers of Demand (Source : Transpower)

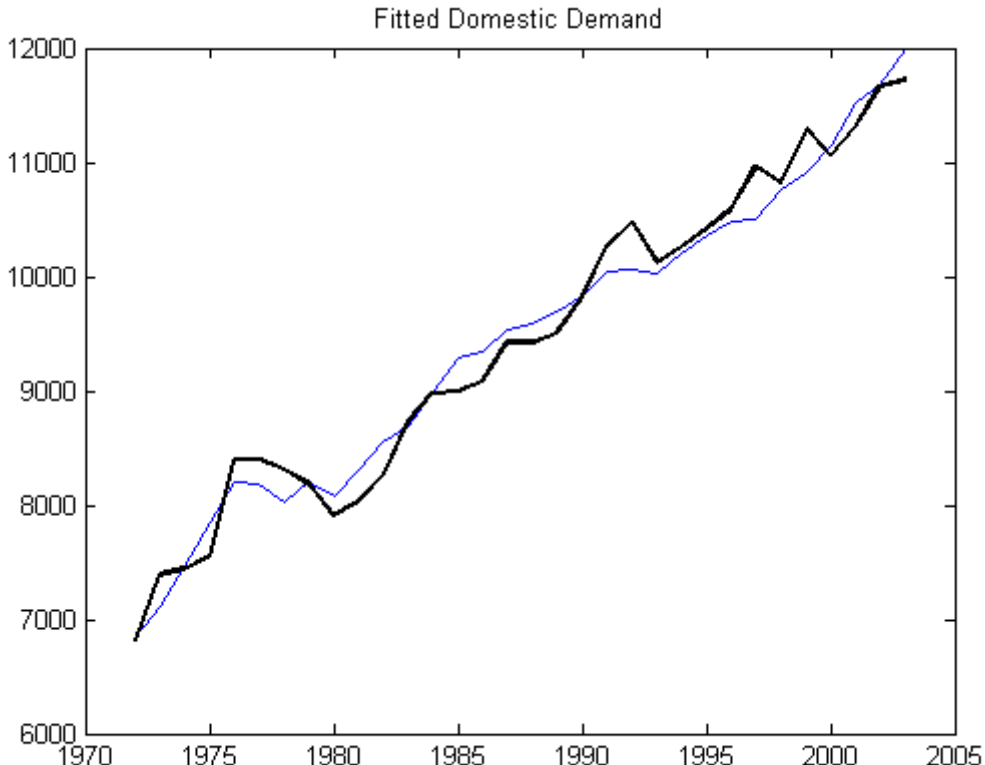
Key Driver	Contributing Factors
GDP	<p>Human resource skill base (enhanced by investment in human capital)</p> <p>Capital intensity</p> <p>Labour force growth</p> <p>Labour force participation</p> <p>Technological change</p> <p>Economic flexibility (i.e. ability of capital and labour to move to their highest value uses)</p> <p>Trade agreements (widening or closing overseas markets) and other international alliances</p> <p>Attractiveness to investors (including existing and planned infrastructure, perceived financial and political stability)</p> <p>Public attitude to business</p> <p>Political and economic policies</p> <p>Ability to adapt to long term weather impacts on GDP</p> <p>International market conditions – particularly key markets such as aluminium, dairy, raw and finished wood products</p> <p>Relative international competitiveness – success of “Brand New Zealand” and relative long term exchange rates</p> <p>Acceptance and adoption of new technologies</p>
Demographics	<p>Number of people – immigration/emigration rates, birth/death rates, age structure, attractiveness of New Zealand as a place to live (lifestyle, security, infrastructure, medical, overseas economic growth rates (particularly Australia’s rate) vs. NZ growth rate), immigration policies</p> <p>Location within New Zealand – local infrastructure, rural depopulation/re-population, regional development policies, age structure (i.e. no. of retirees)</p>
Electricity Prices	<p>Resource costs – gas/coal/other and availability of infrastructure effects on generation, potential carbon charges</p> <p>Effect of prices on investment in energy-intensive industries</p> <p>Pricing policies – fixed /variable mix and resulting incentives on consumers</p> <p>Substitution with other energy sources – people switching to direct use of the alternative fuel depending on availability and cost</p> <p>Quality of electricity demanded (affecting cost of underlying infrastructure)</p> <p>Metering technologies</p>

<p>Embedded Generation</p>	<p>Large generator and lines company behaviour and incentives – e.g. facilitation of small generators injecting into local networks, large wind vs. small wind</p> <p>Technology price changes – fuel cells, cheap solar vs. changes in cost of large scale generation</p> <p>Manufacturing industry types – some industries more suited to co-generation</p> <p>Quality standard of grid supplied electricity compared to consumer requirements – need for backup plant</p> <p>Resource availability – gas availability, bio-mass (impact of forestry development)</p> <p>Central government policies – Resource Management Act</p> <p>Existing infrastructure (e.g. proximity of existing grid assets, gas lines)</p>
<p>Energy Intensity</p>	<p>Technology – improving efficiency and improving quality of housing stock vs. new electricity consuming items (e.g. electric cars)</p> <p>Public attitude and expectations – social “greenness” (frugality vs. comfort expectations)</p> <p>Environmental policy requirements – political structures and public impact on policies</p> <p>Changes in industrial and commercial types (i.e. economic structure) – industries changing across types (e.g. evolving to service industries) and within types (e.g. dairy farming replacing other farming and raw vs. finished wood products)</p> <p>Weather</p>

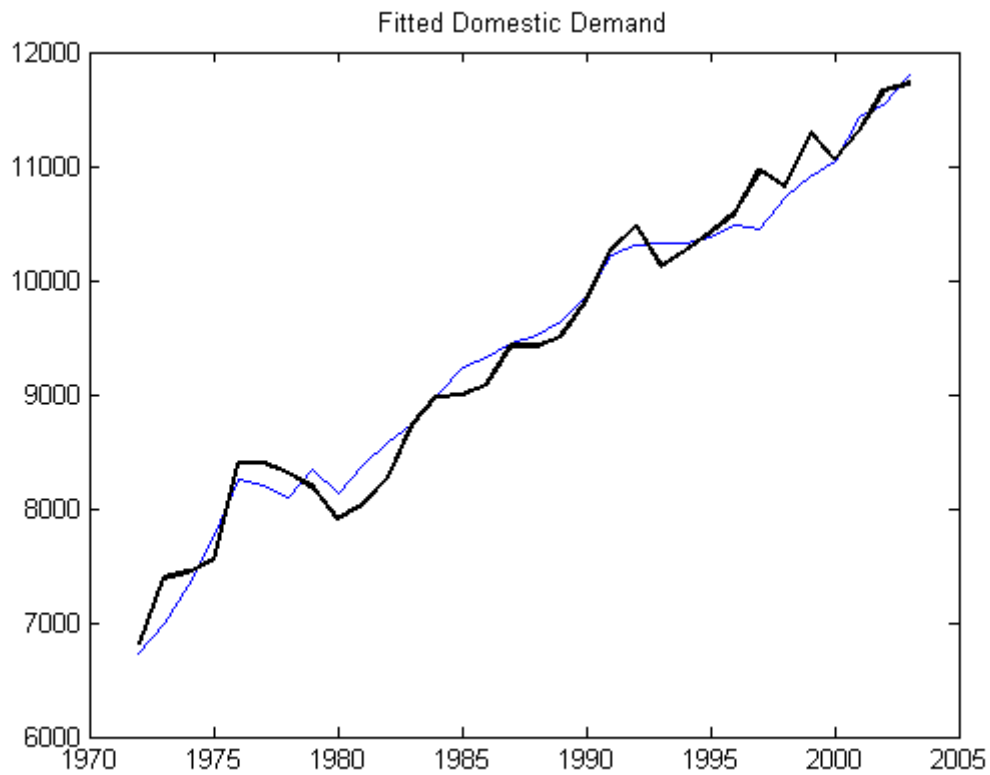
Appendix B: Comparison of Residential Models



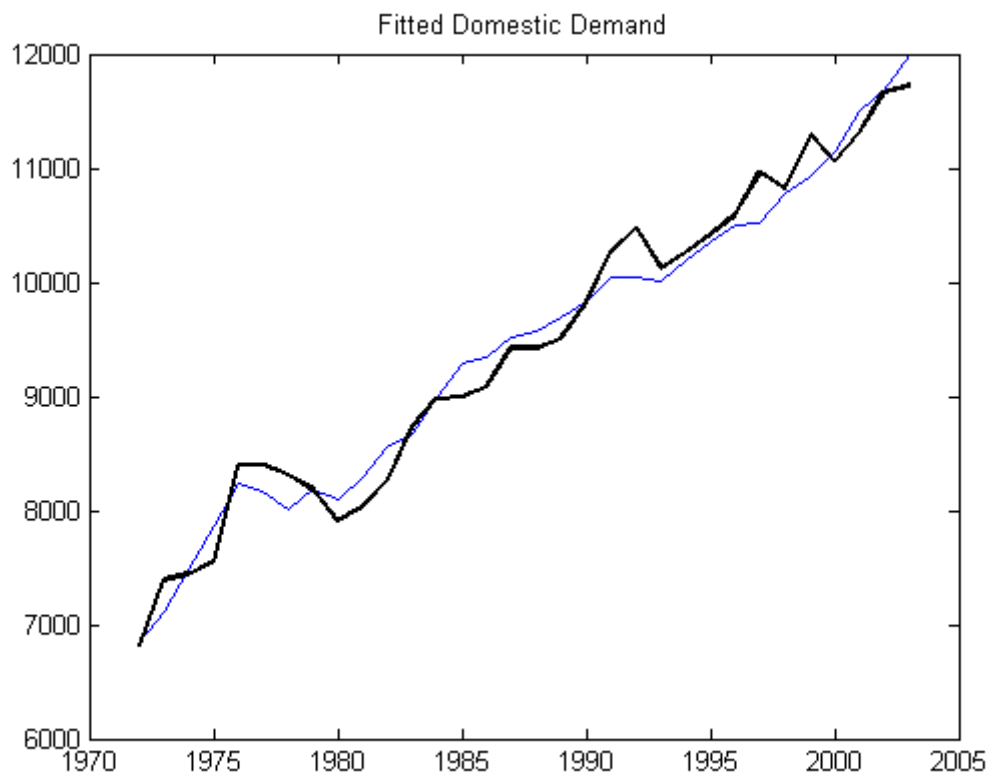
Two Stage Log V1 (Transpower) – Actual in black



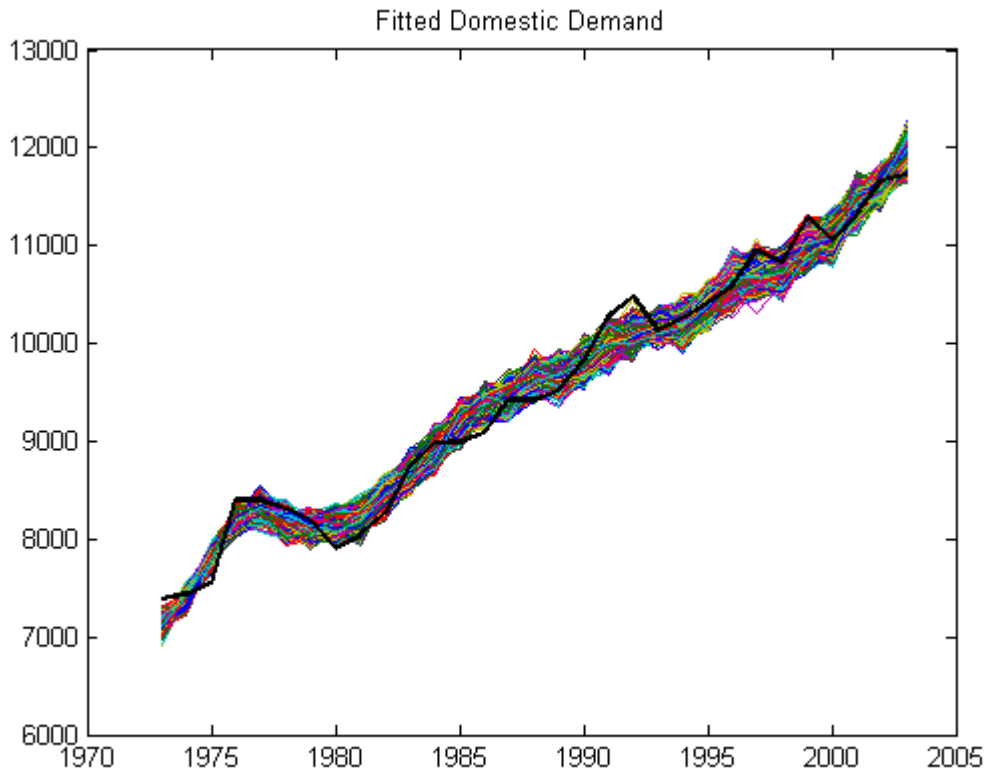
Single Stage Linear V2 – Actual in black



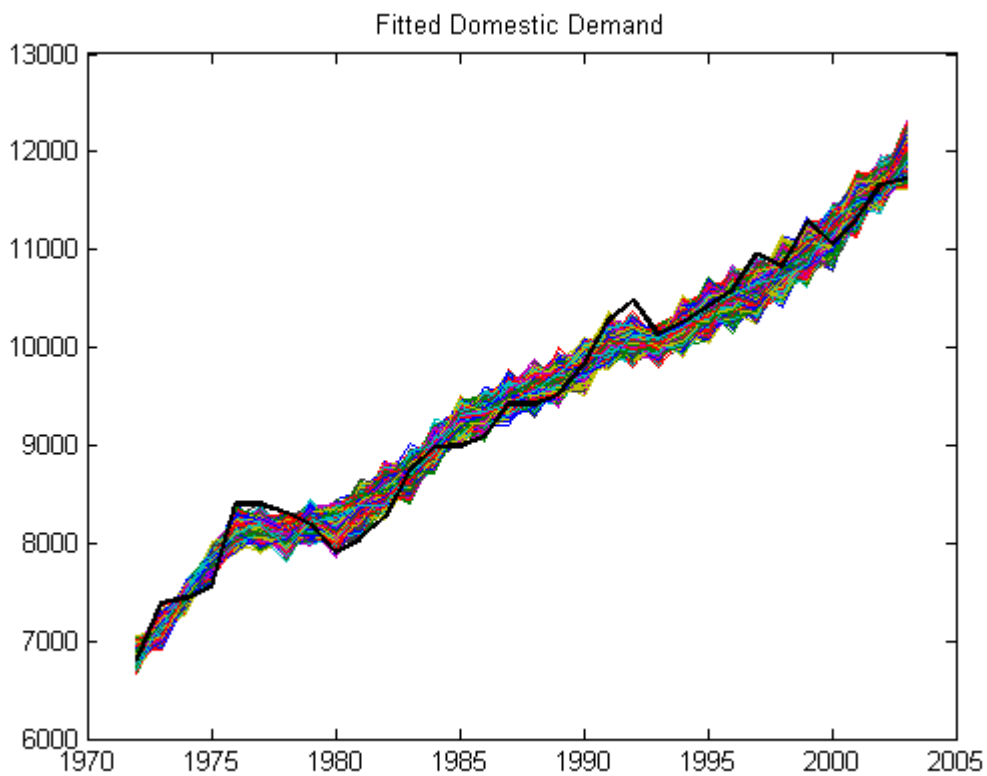
Single Stage Log V1 – Actual in black



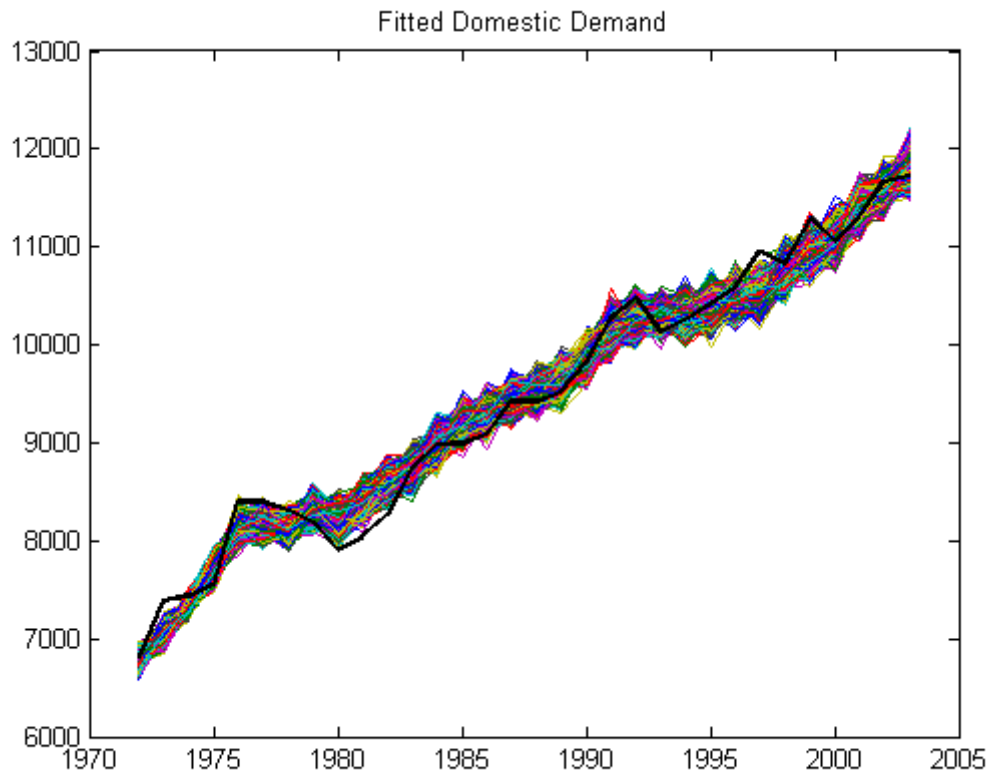
Single Stage Log V2 – Actual in black



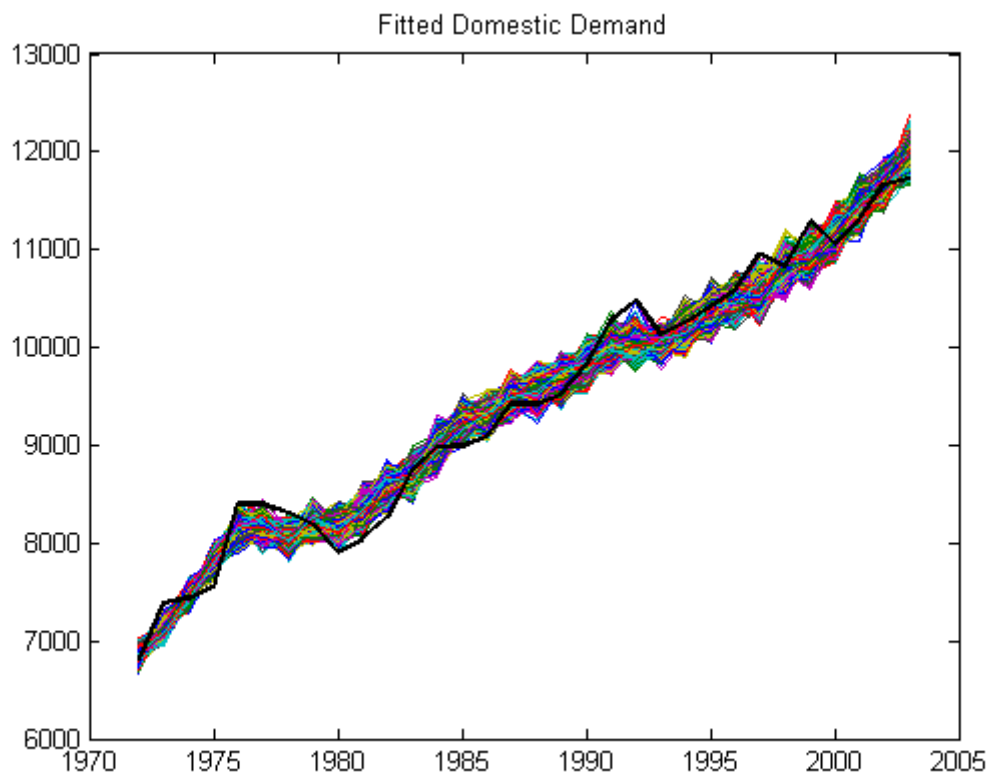
Two Stage Log V1 (Transpower) – Input variation Monte Carlo 10000 runs



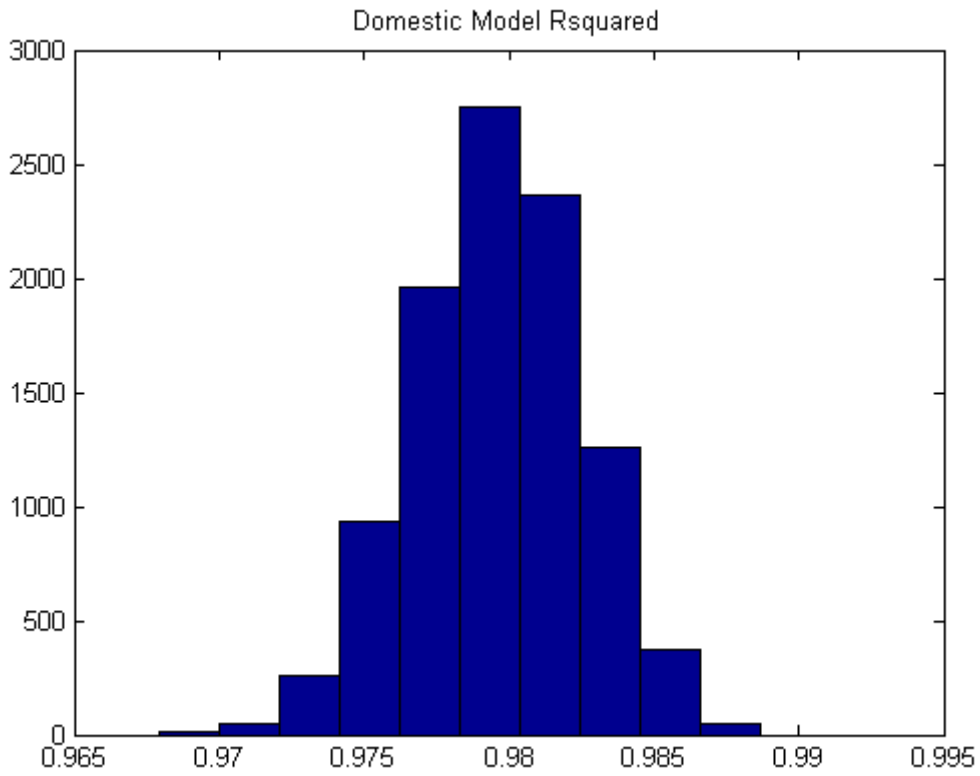
Single Stage Linear V2 – Input variation Monte Carlo 10000 runs



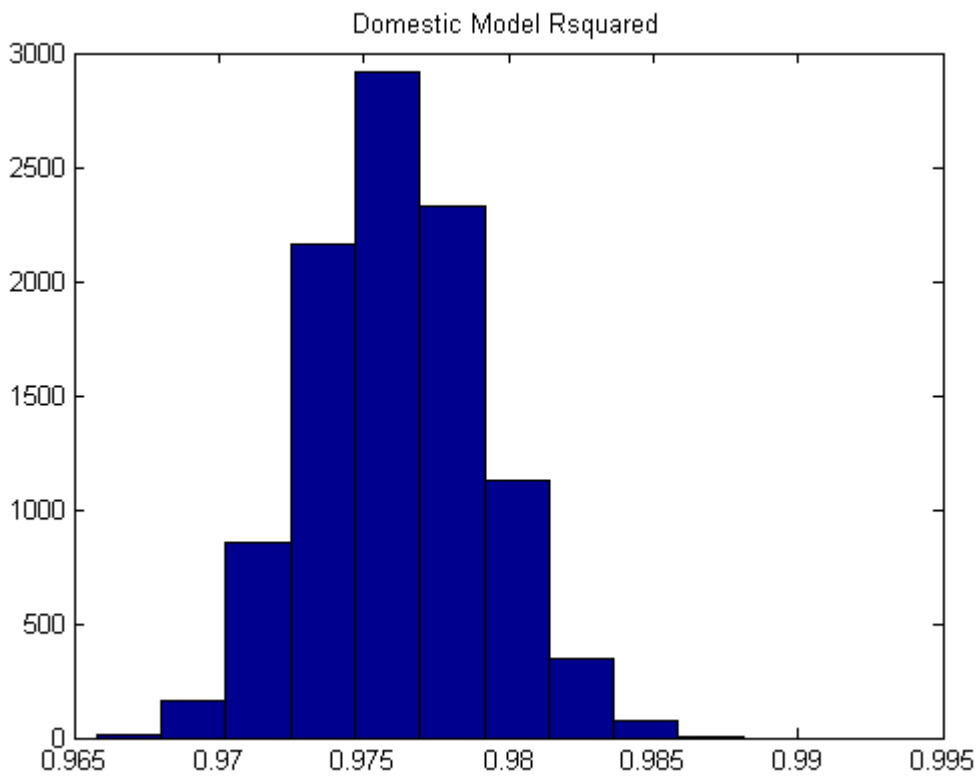
Single Stage Log V1 – Input variation Monte Carlo 10000 runs



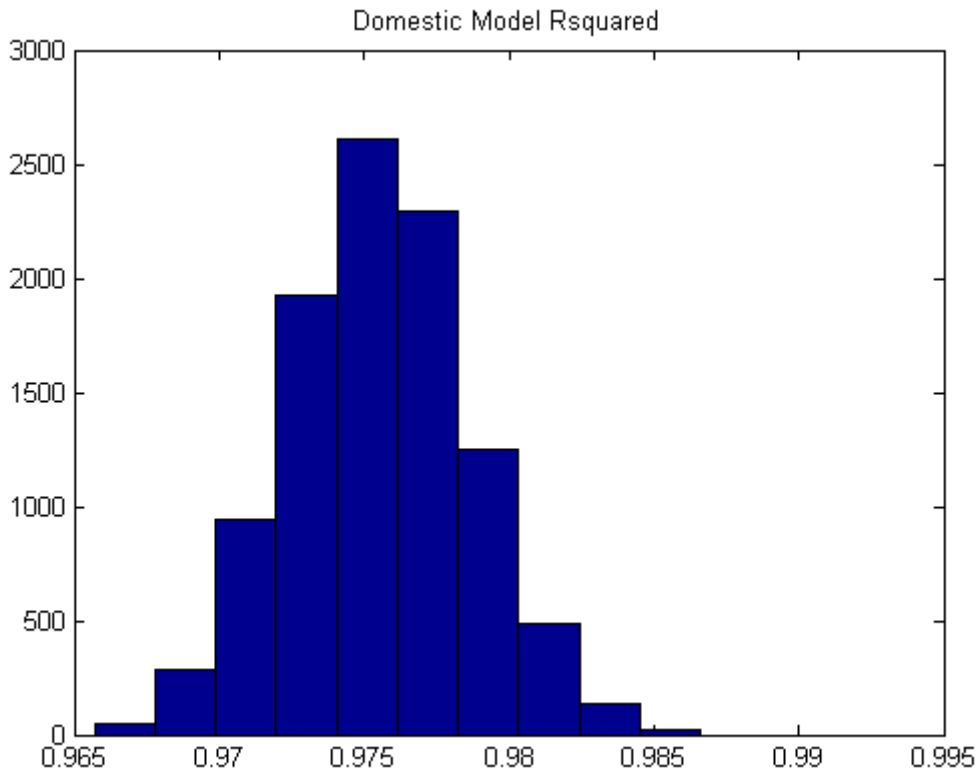
Single Stage Log V2 – Input variation Monte Carlo 10000 runs



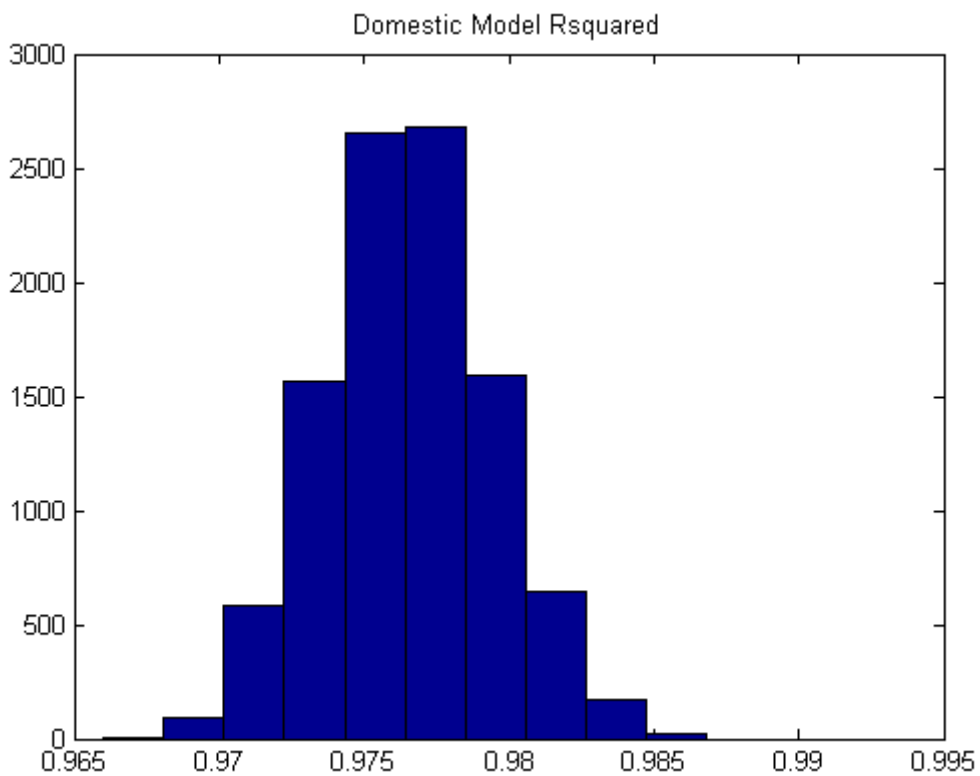
Two Stage Log V1 (Transpower) – Input variation Monte Carlo 10000 runs



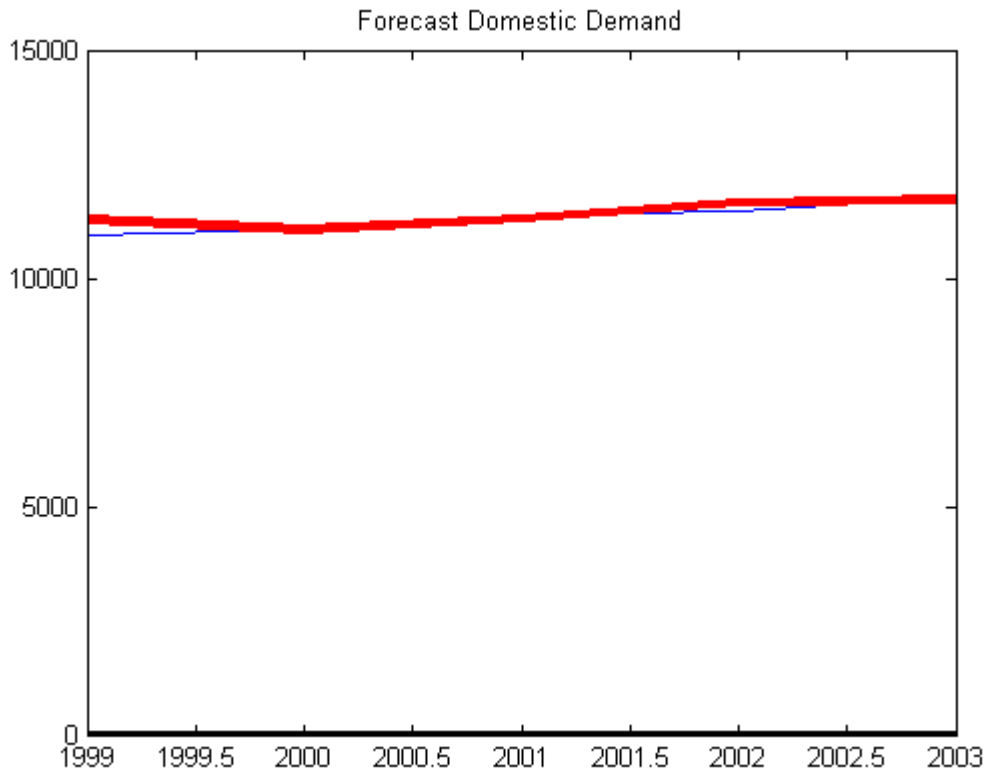
Single Stage Linear V2 – Input variation Monte Carlo 10000 runs



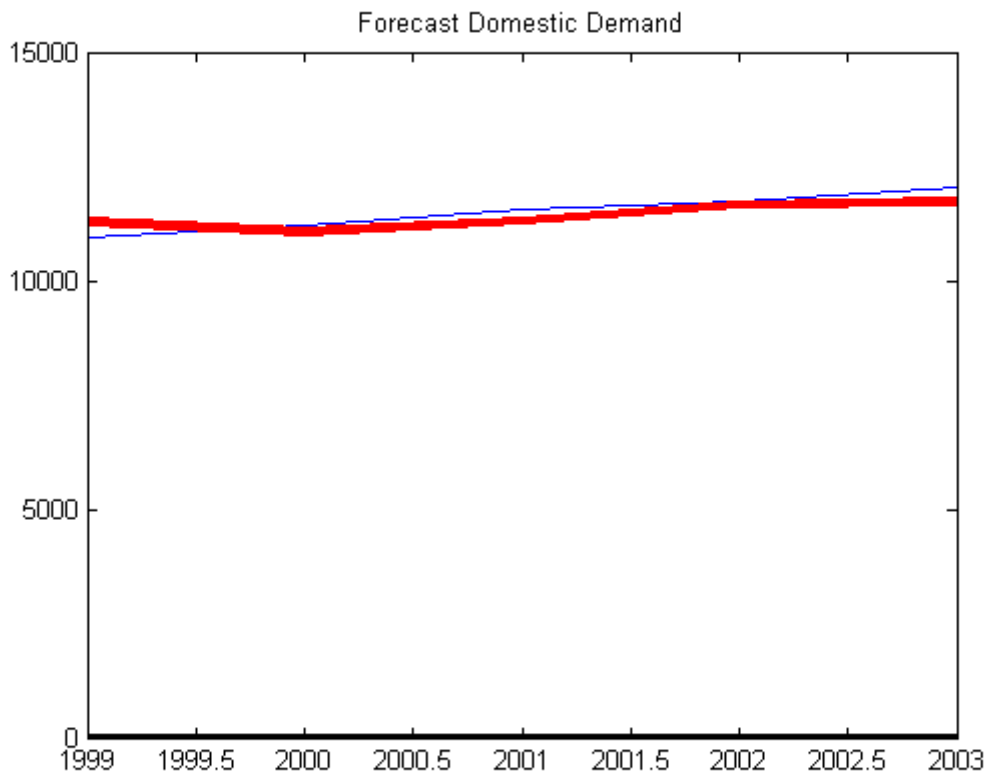
Single Stage Log V1 – Input variation Monte Carlo 10000 runs



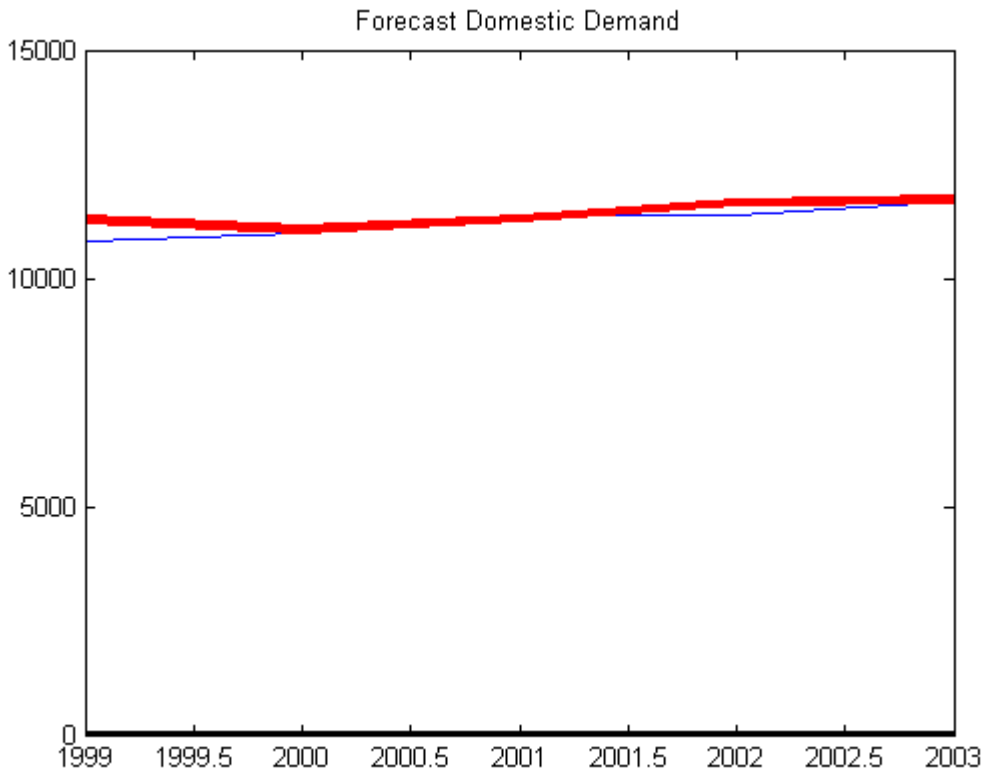
Single Stage Log V2 – Input variation Monte Carlo 10000 runs



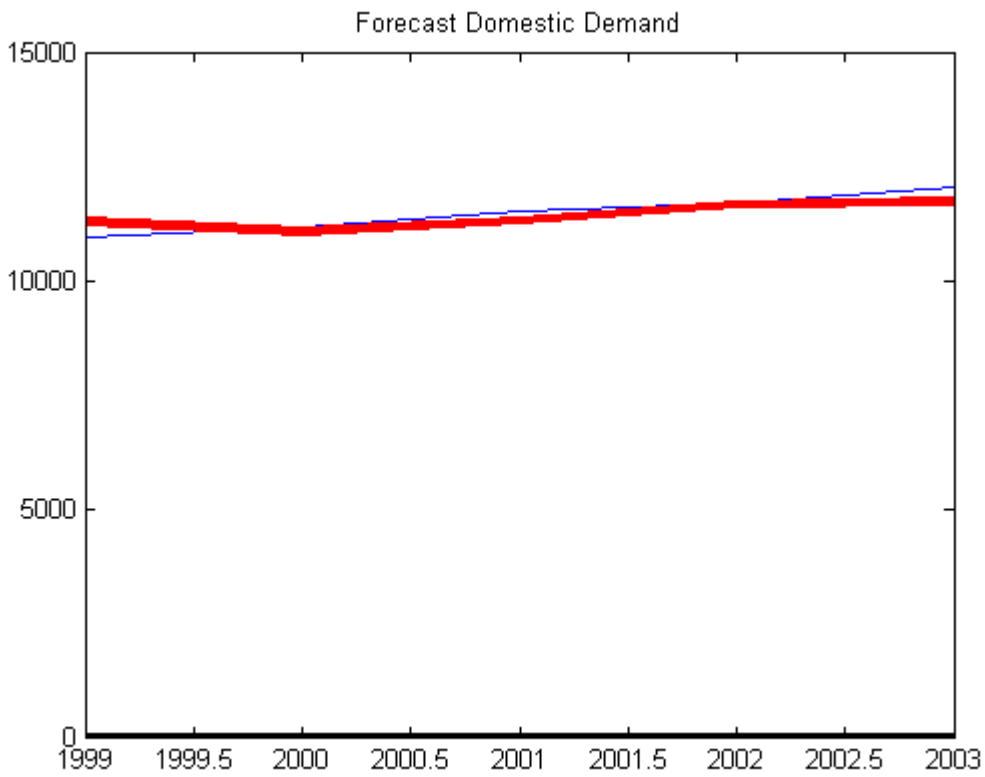
Two Stage Log V1 (Transpower) – Truncated Forecast (Actual in Red)



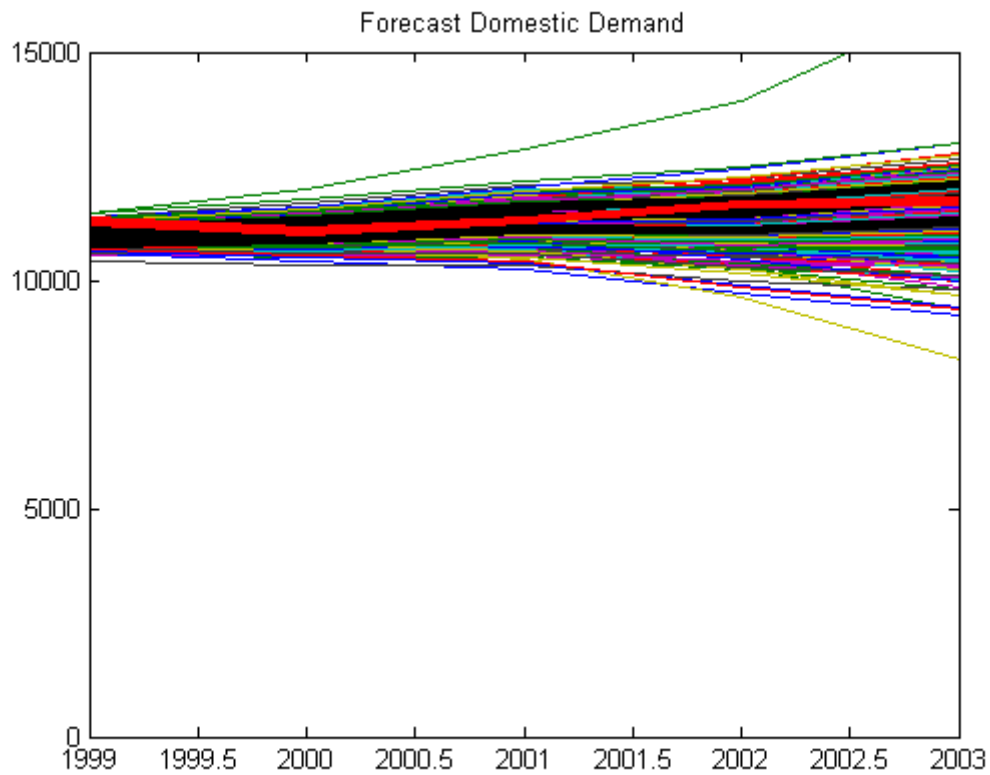
Single Stage Linear V1 – Truncated Forecast (Actual in Red)



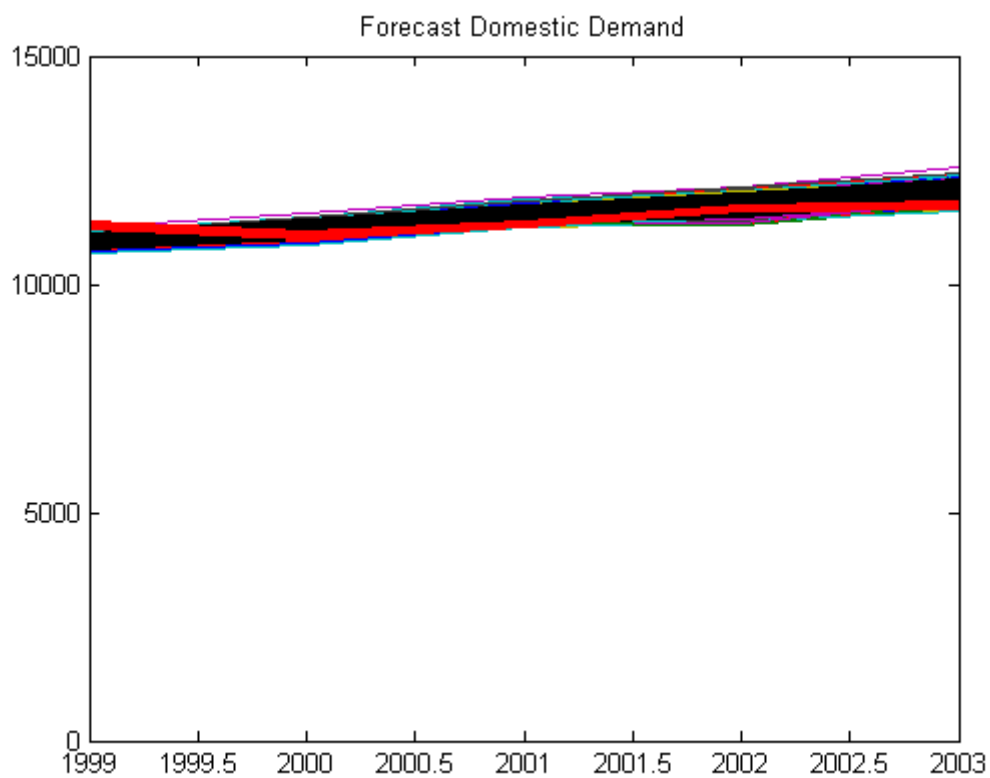
Single Stage Log V1 – Truncated Forecast (Actual in Red)



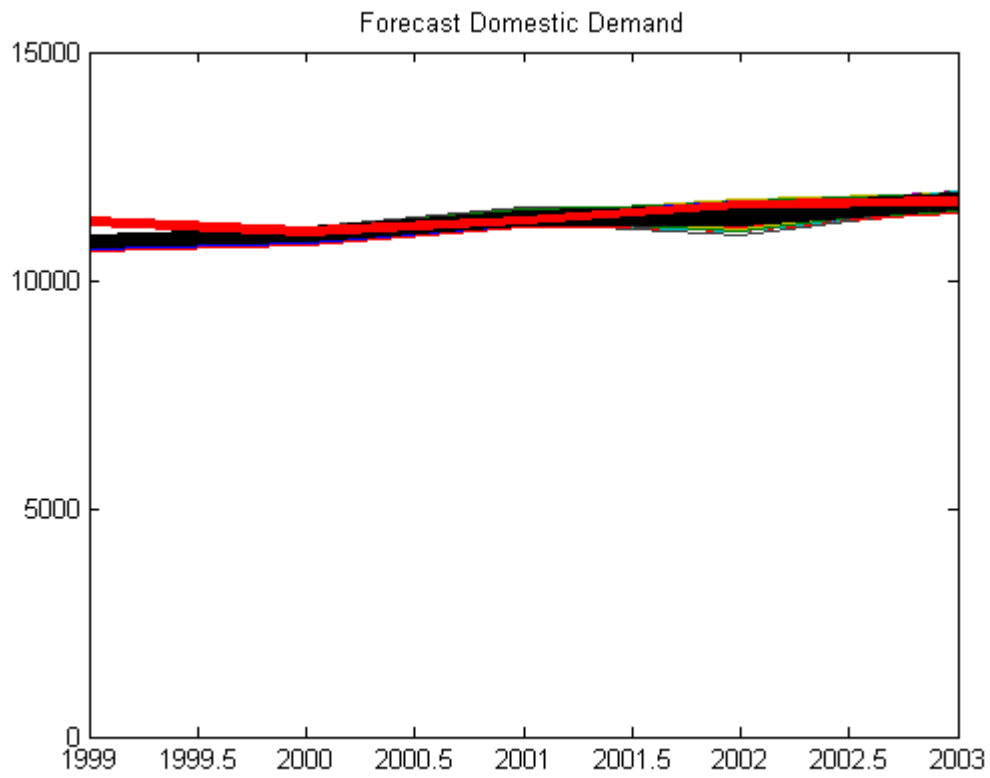
Single Stage Log V2 – Truncated Forecast (Actual in Red)



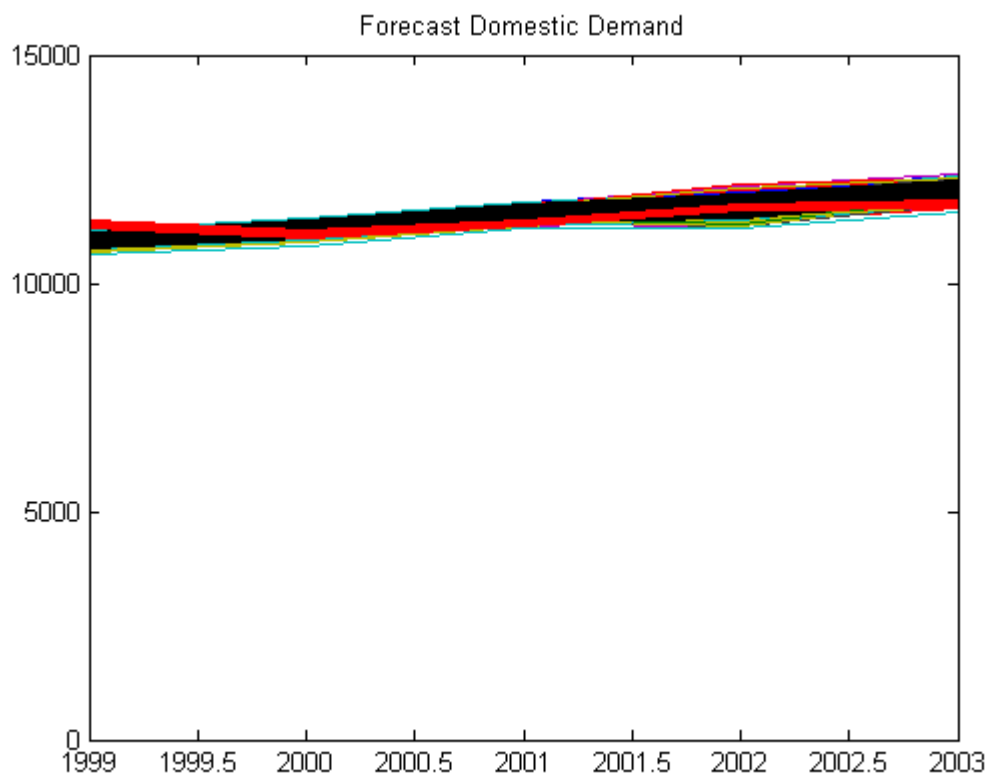
Two Stage Log V1 (Transpower) - Monte Carlo 10000 runs (Actual in Red)



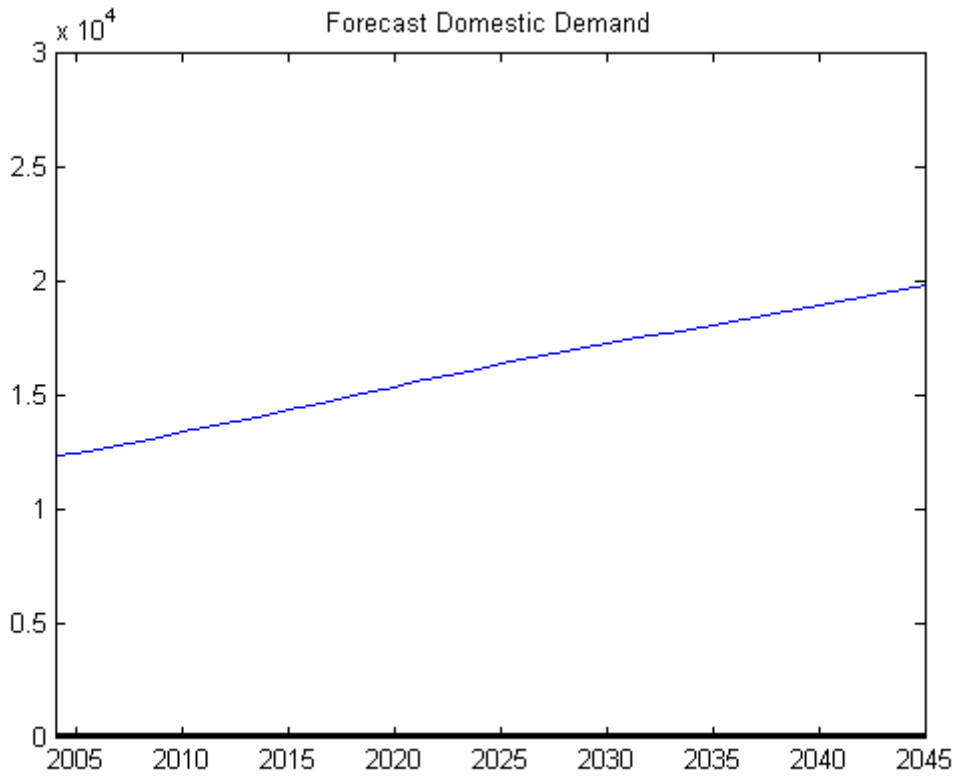
Single Stage Linear V2 - Monte Carlo 10000 runs (Actual in Red)



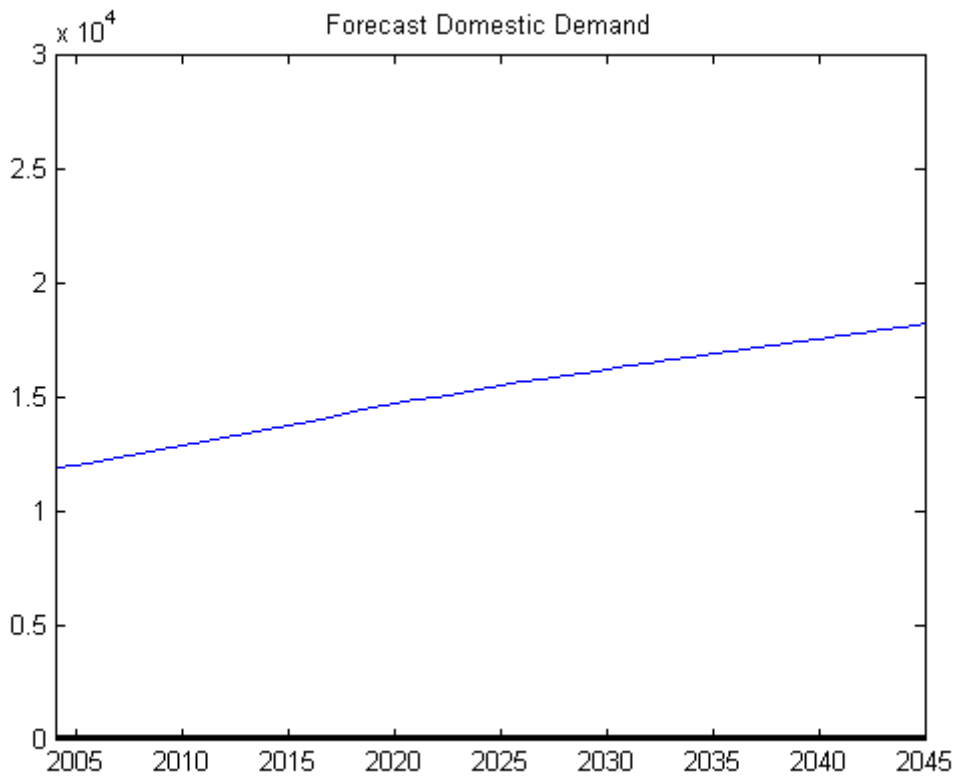
Single Stage LogV1 - Monte Carlo 10000 runs (Actual in Red)



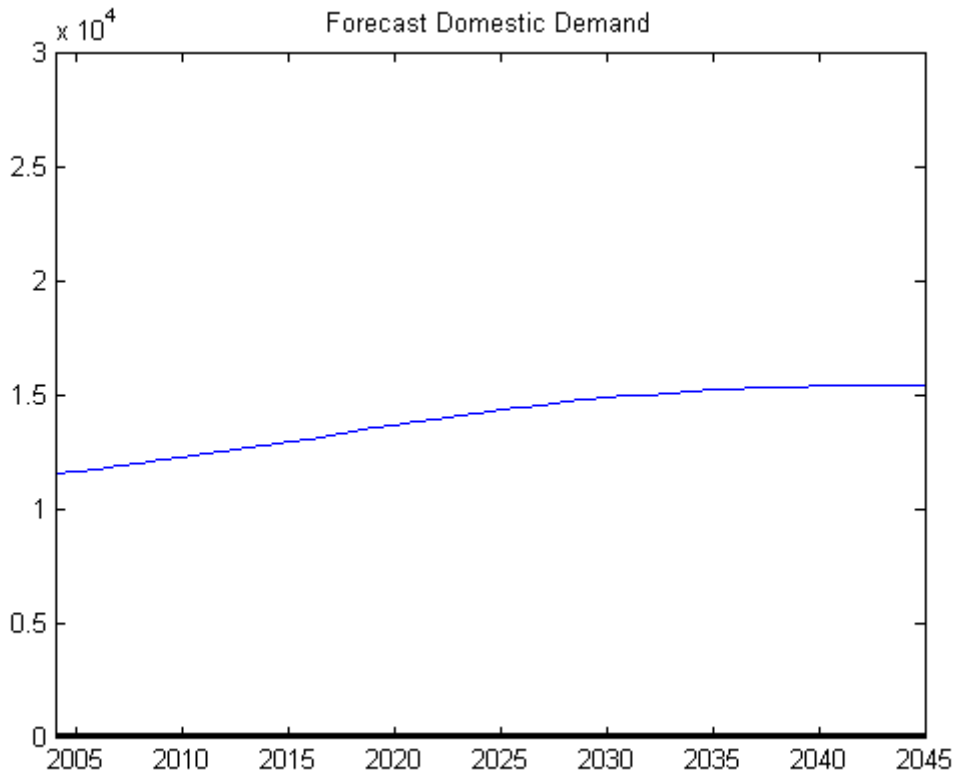
Single Stage LogV2 - Monte Carlo 10000 runs (Actual in Red)



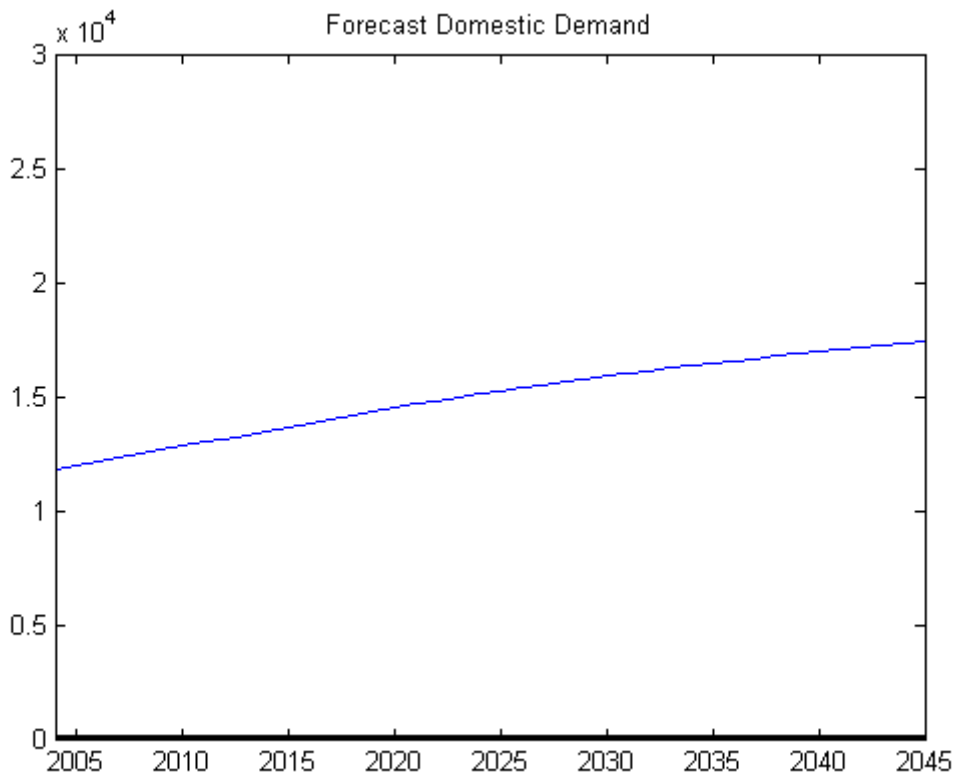
Two Stage Log V1 (Transpower)



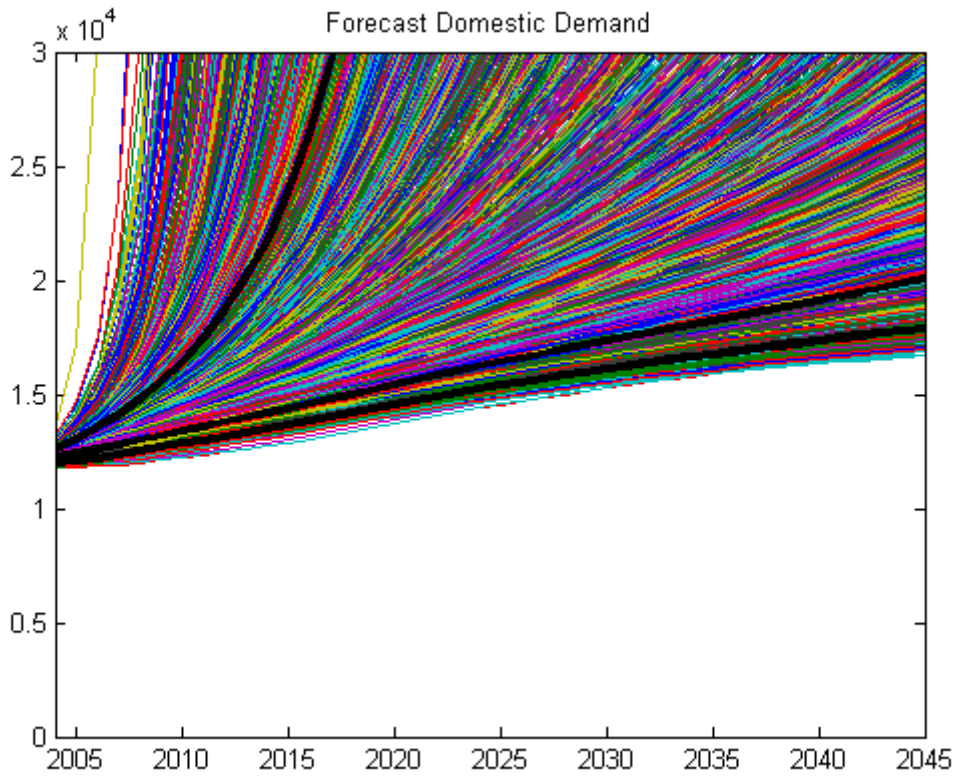
Single Stage Linear V2



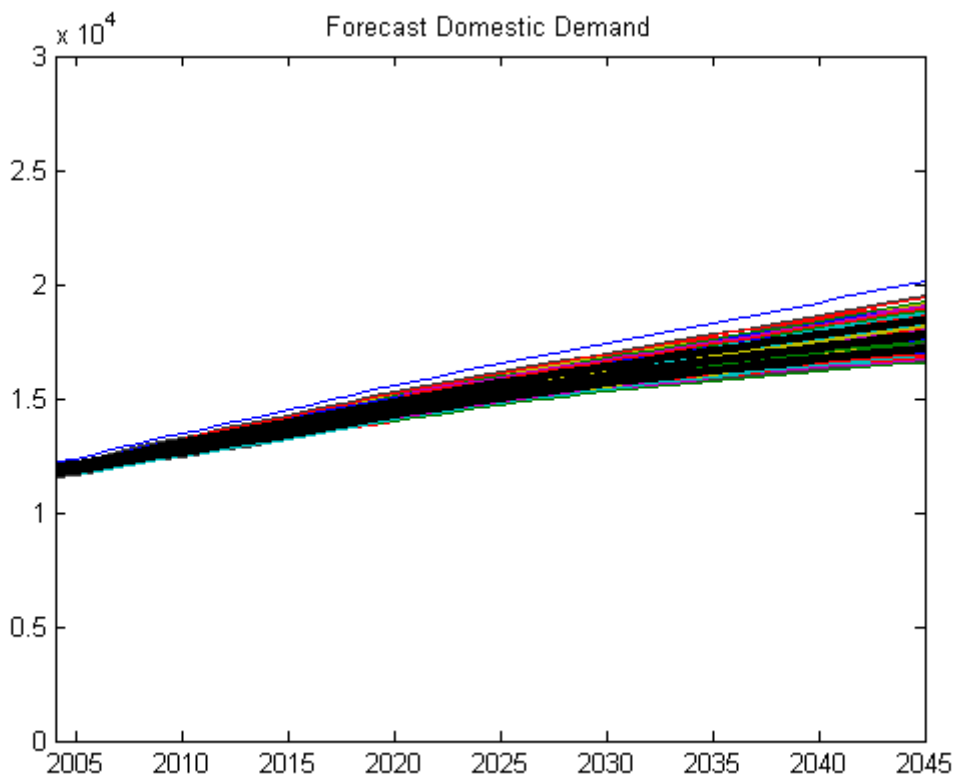
Single Stage LogV1



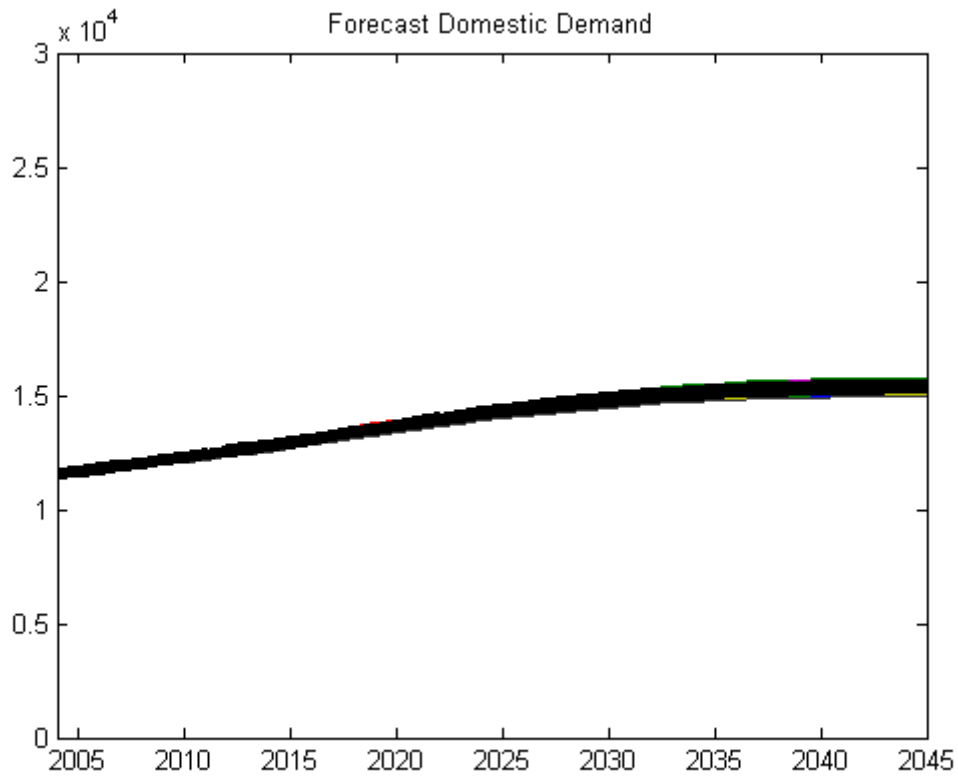
Single Stage LogV2



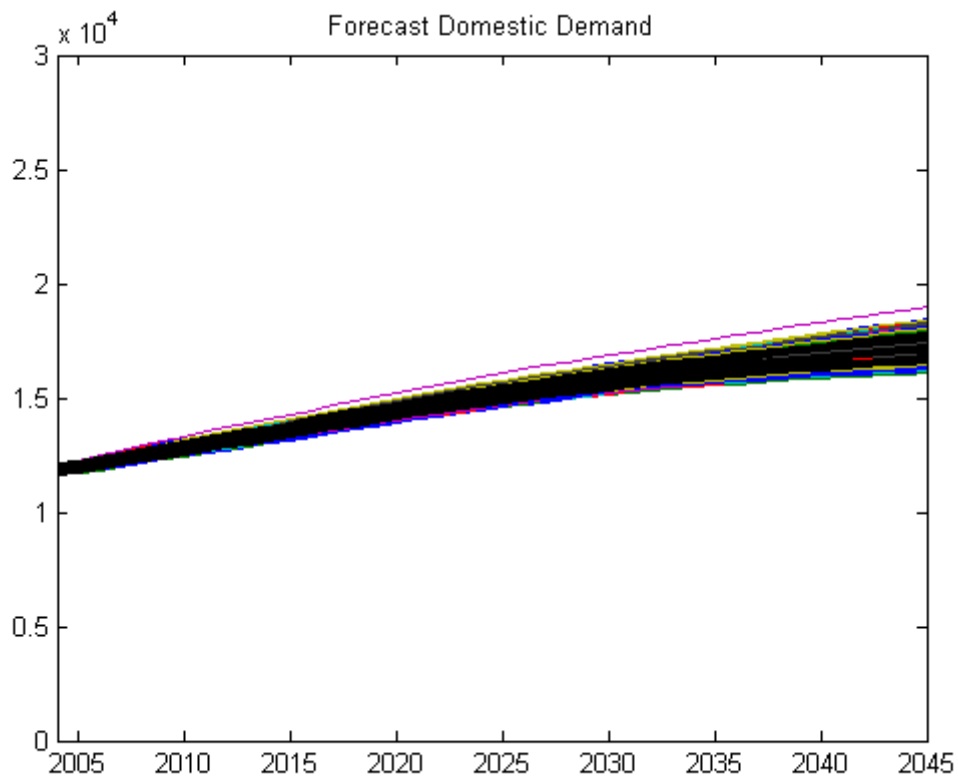
Two Stage Log V1 (Transpower) - Monte Carlo 10000 runs 90 % conf.



Single Stage Linear V2 - Monte Carlo 10000 runs 90 % conf.

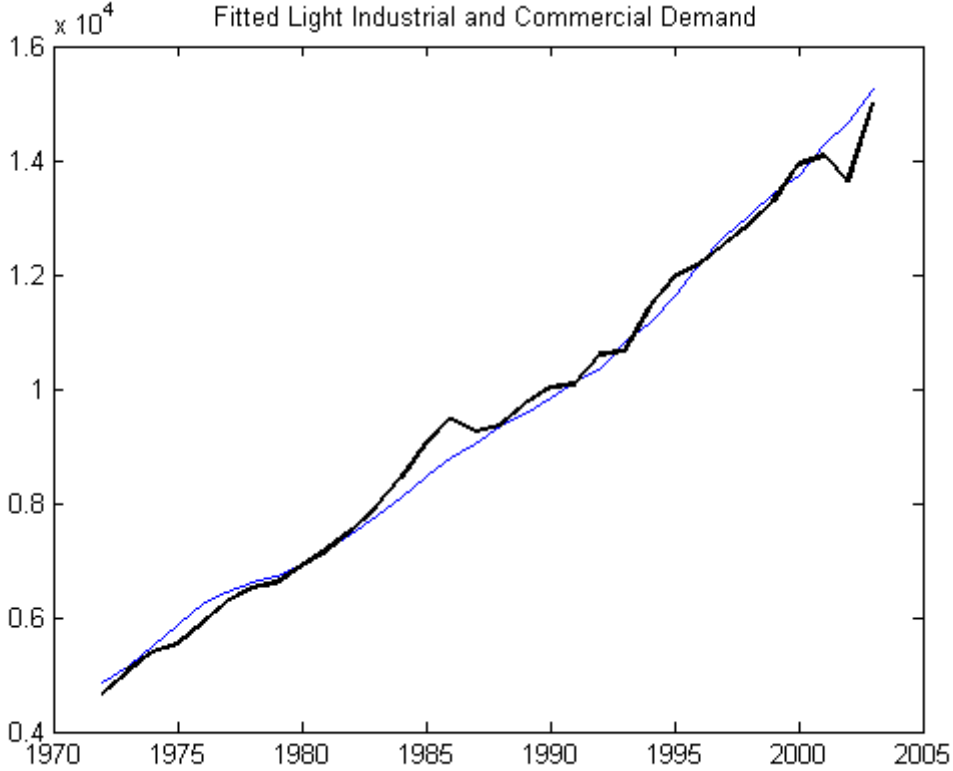


Single Stage Log V1 - Monte Carlo 10000 runs 90 % conf.

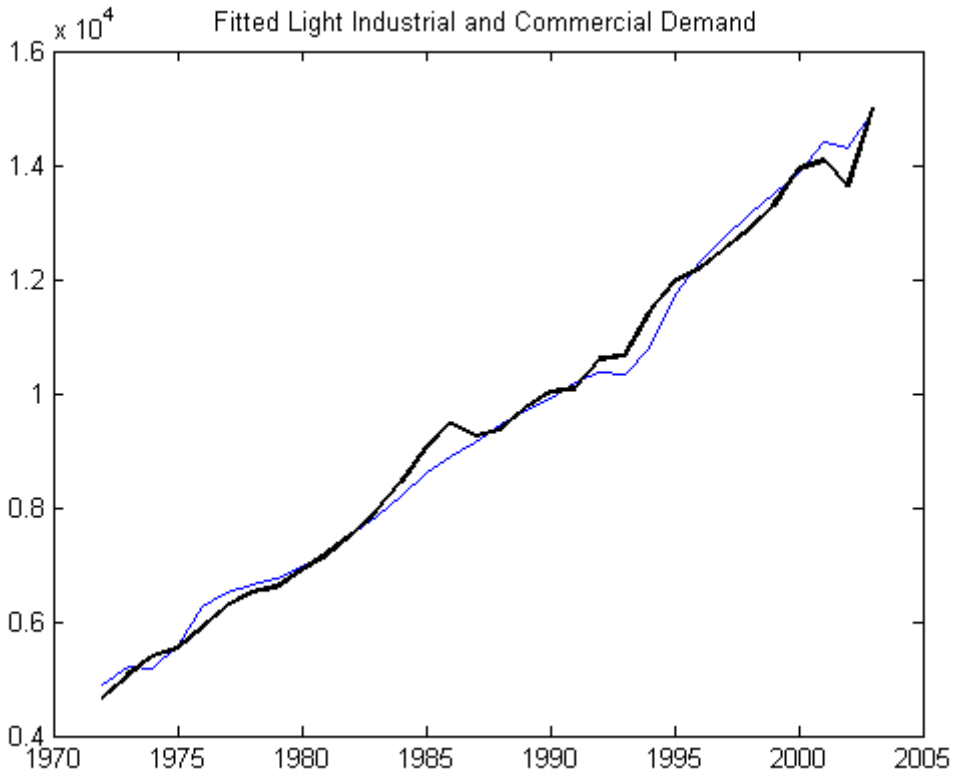


Single Stage Log V2 - Monte Carlo 10000 runs 90 % conf.

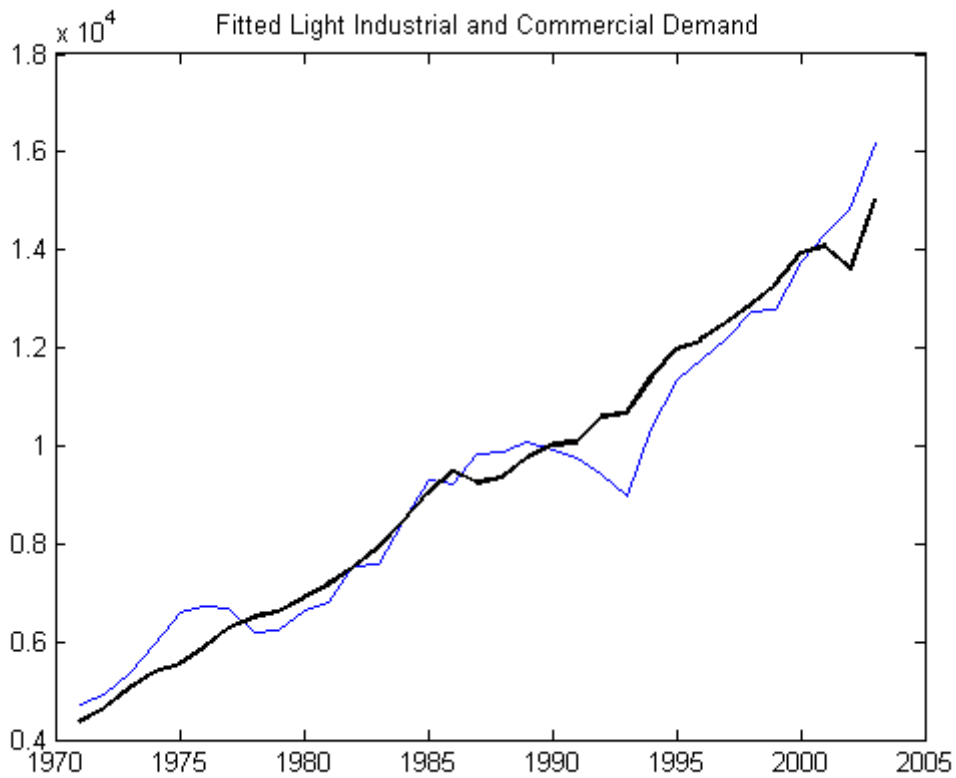
Appendix C: Comparison of Commercial/Light Industrial Models



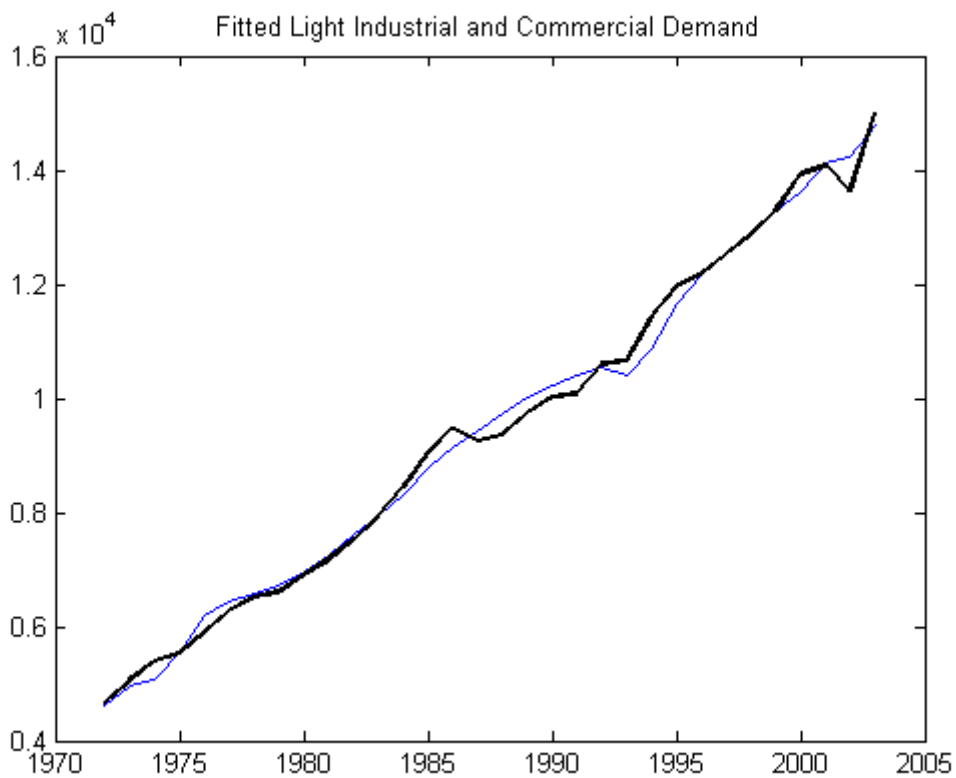
Two Stage Linear V3 (Transpower) – Actual in black



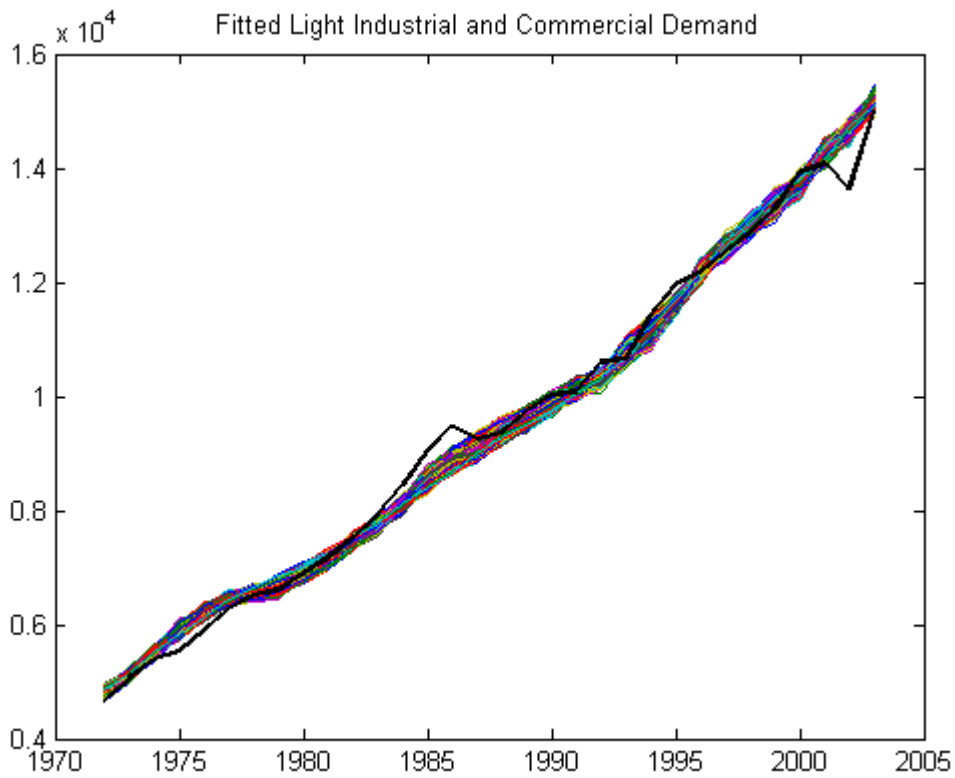
Two Stage Linear V1 – Actual in black



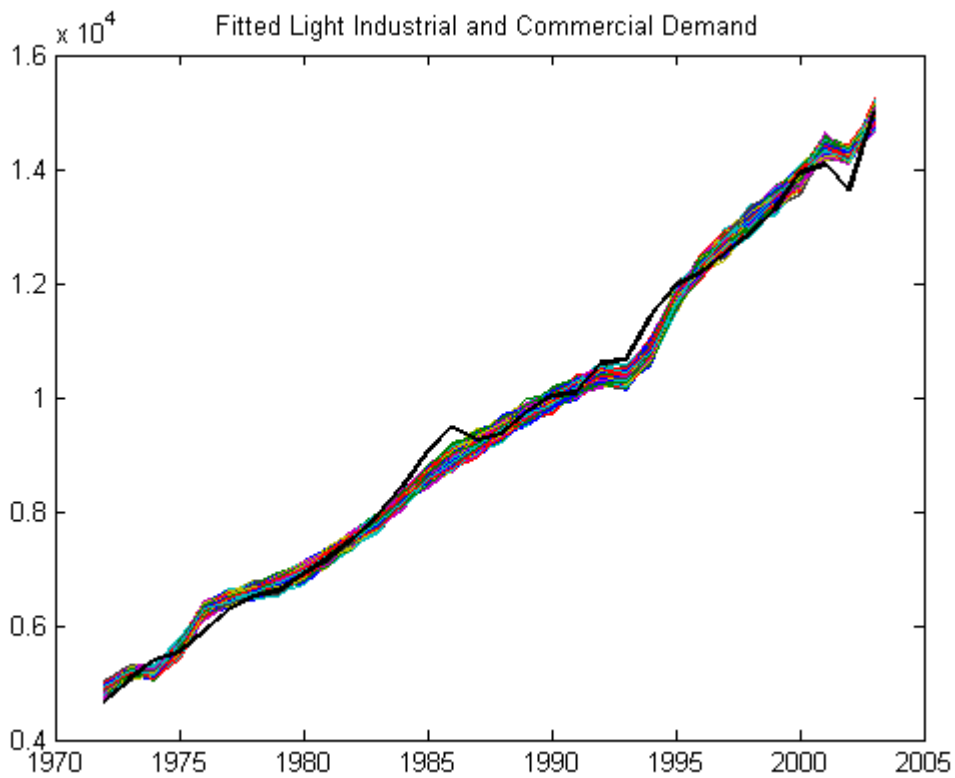
Single Stage Linear V2 – Actual in black



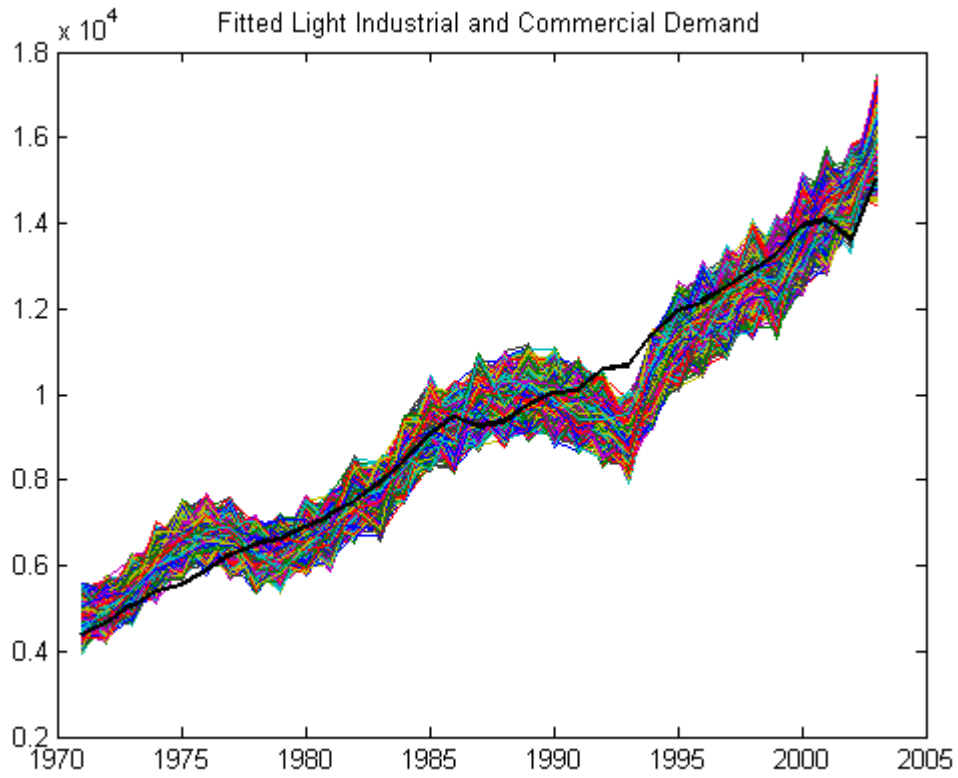
Two Stage Linear V5 – Actual in black



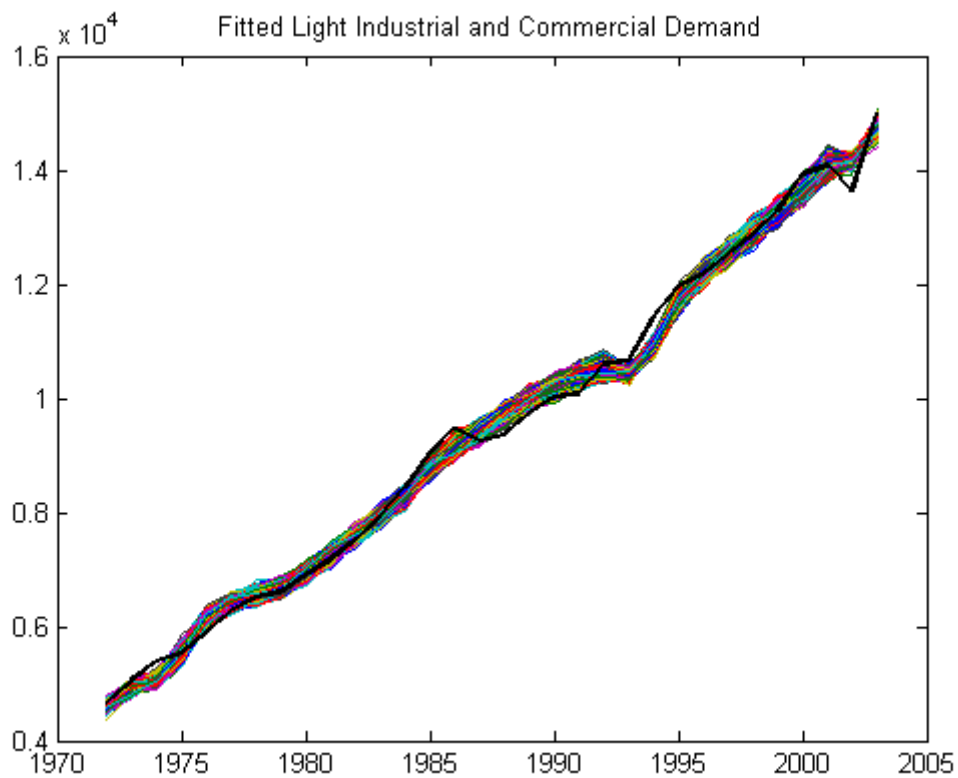
Two Stage Linear V3 (Transpower) – Input variation Monte Carlo 10000 runs



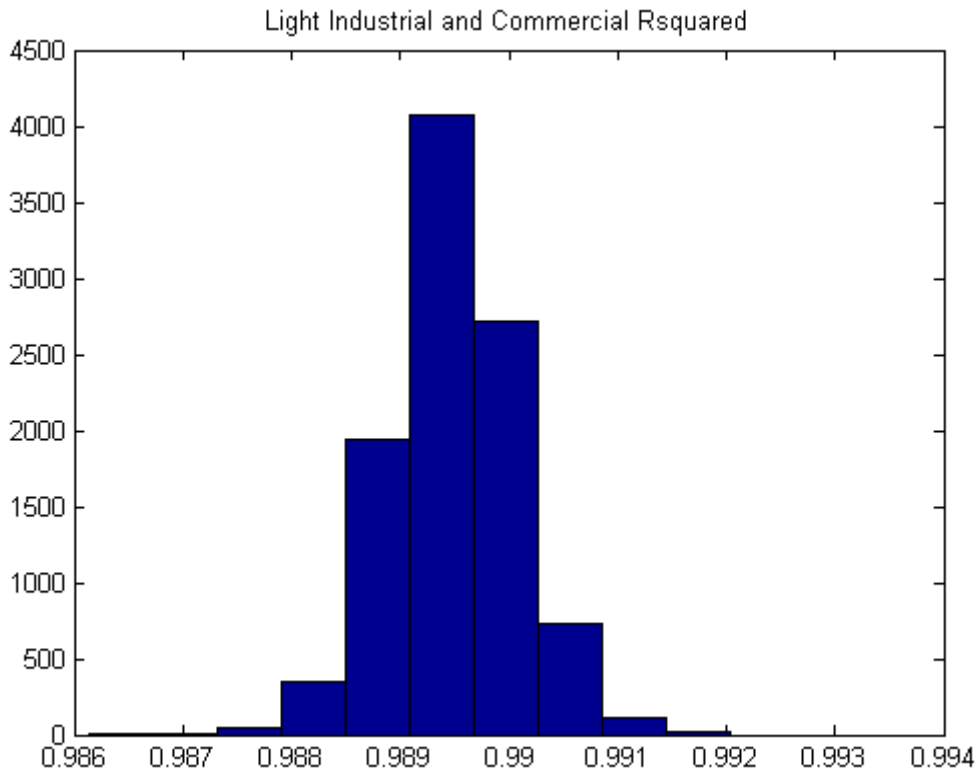
Two Stage Linear V1 – Input variation Monte Carlo 10000 runs



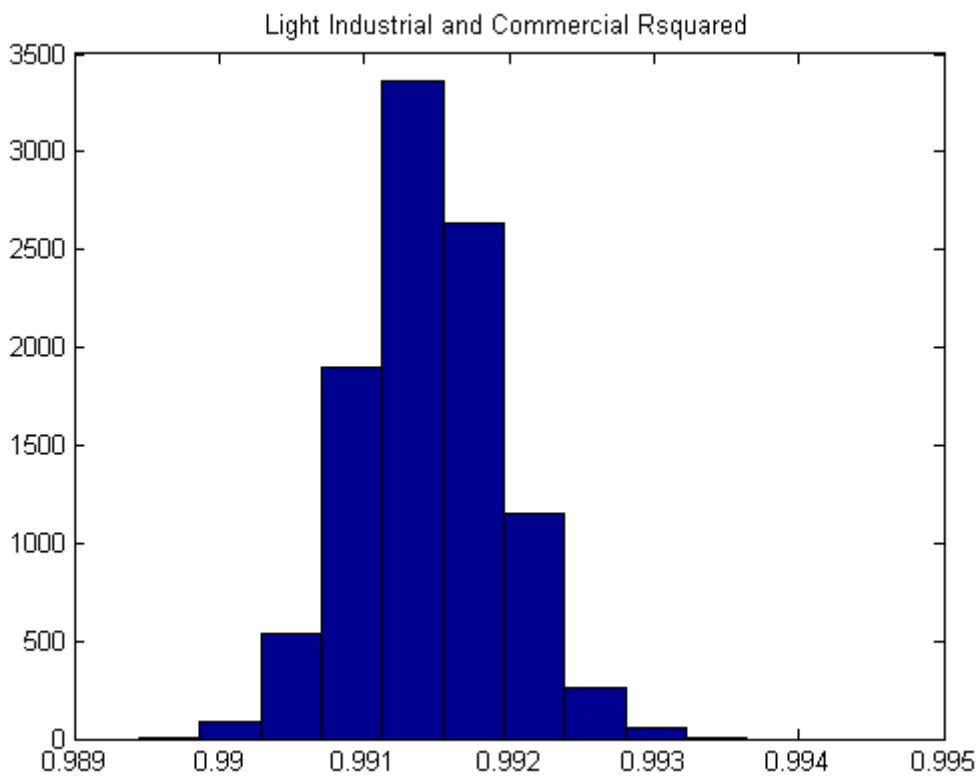
Single Stage Linear V2 – Input variation Monte Carlo 10000 runs



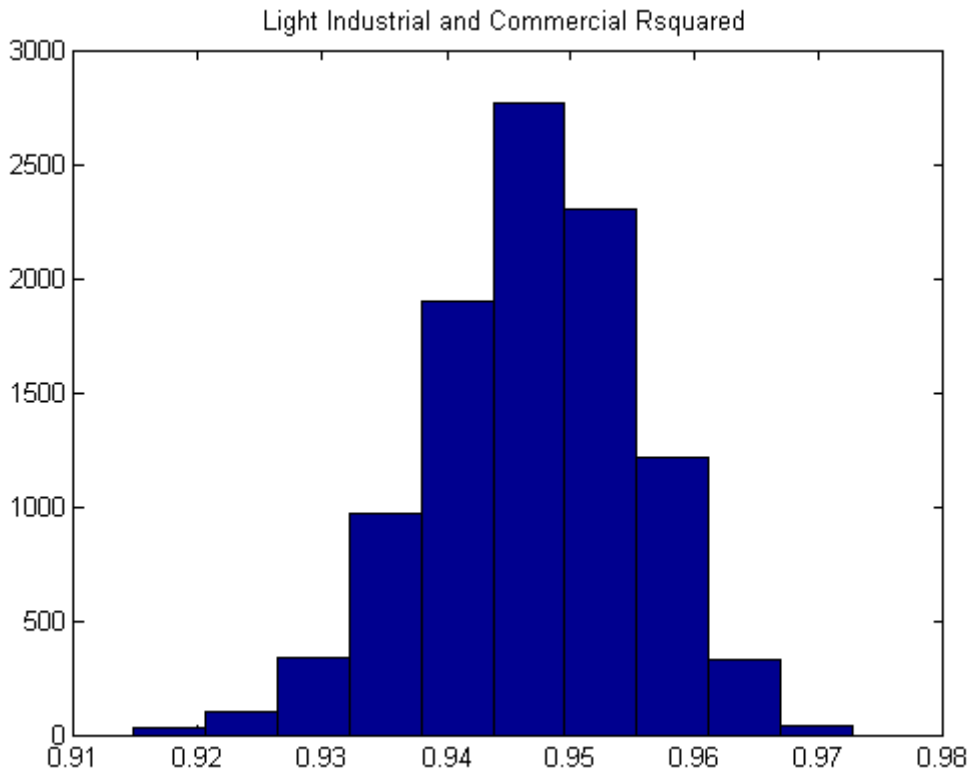
Two Stage Linear V5 – Input variation Monte Carlo 10000 runs



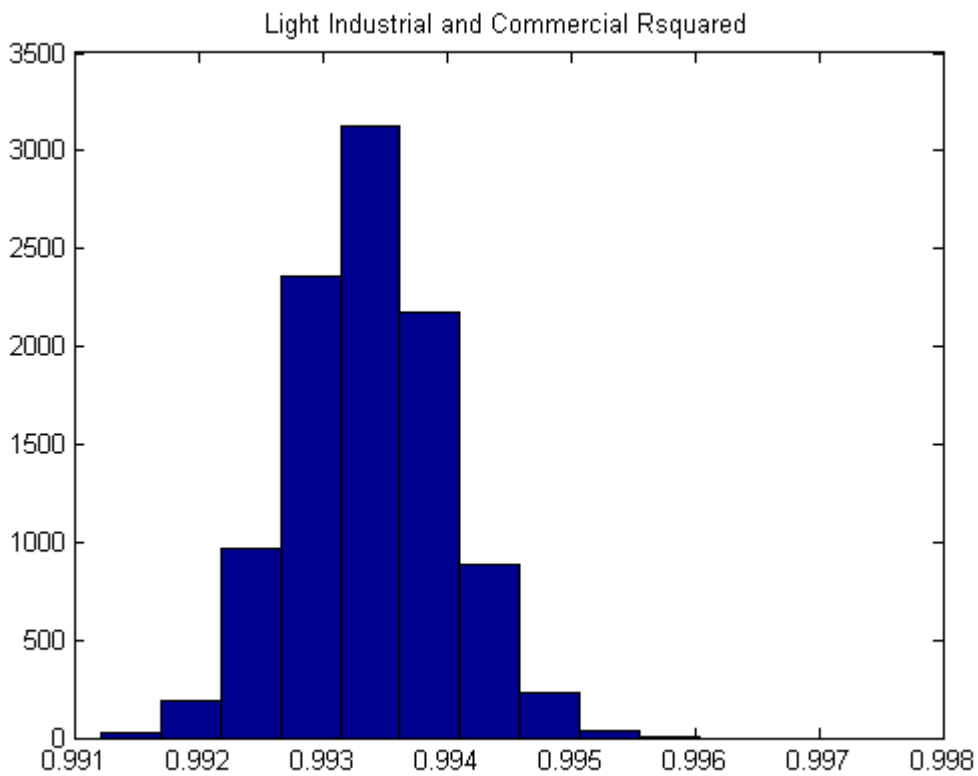
Two Stage Linear V3 (Transpower) – Input variation Monte Carlo 10000 runs



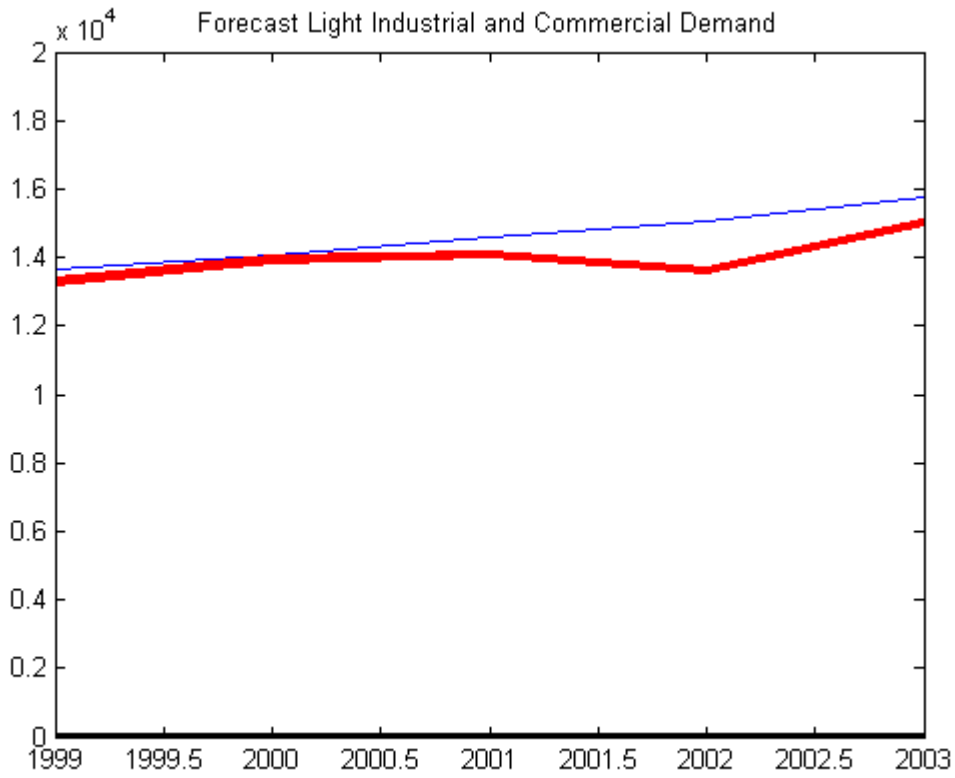
Two Stage Linear V3 – Input variation Monte Carlo 10000 runs



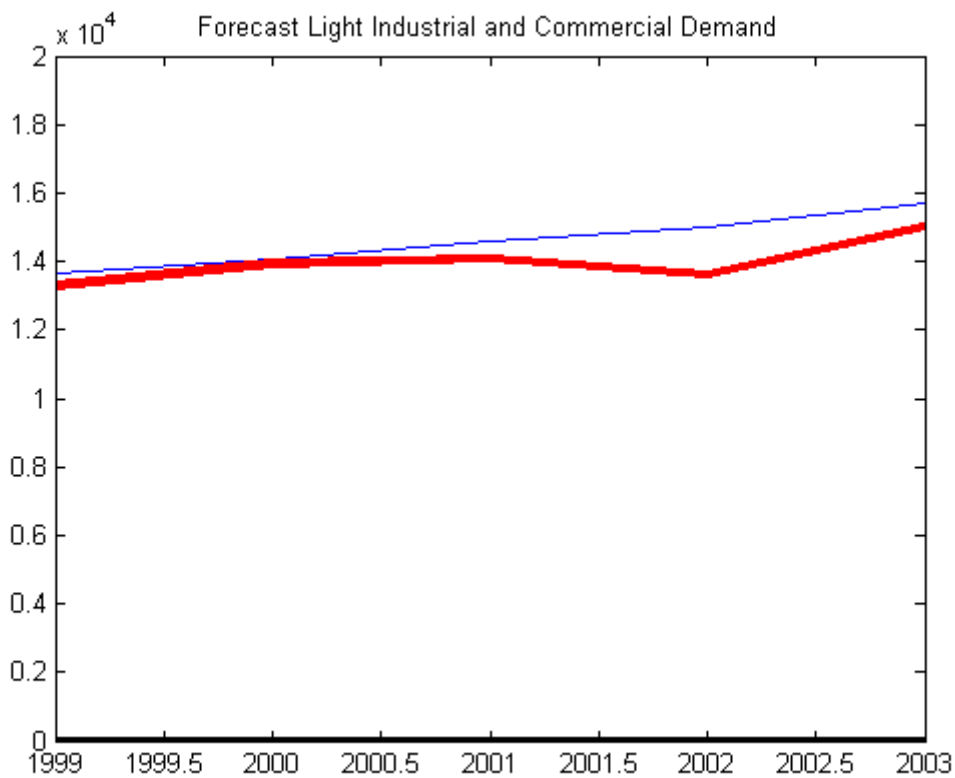
Single Stage Linear V2 – Input variation Monte Carlo 10000 runs



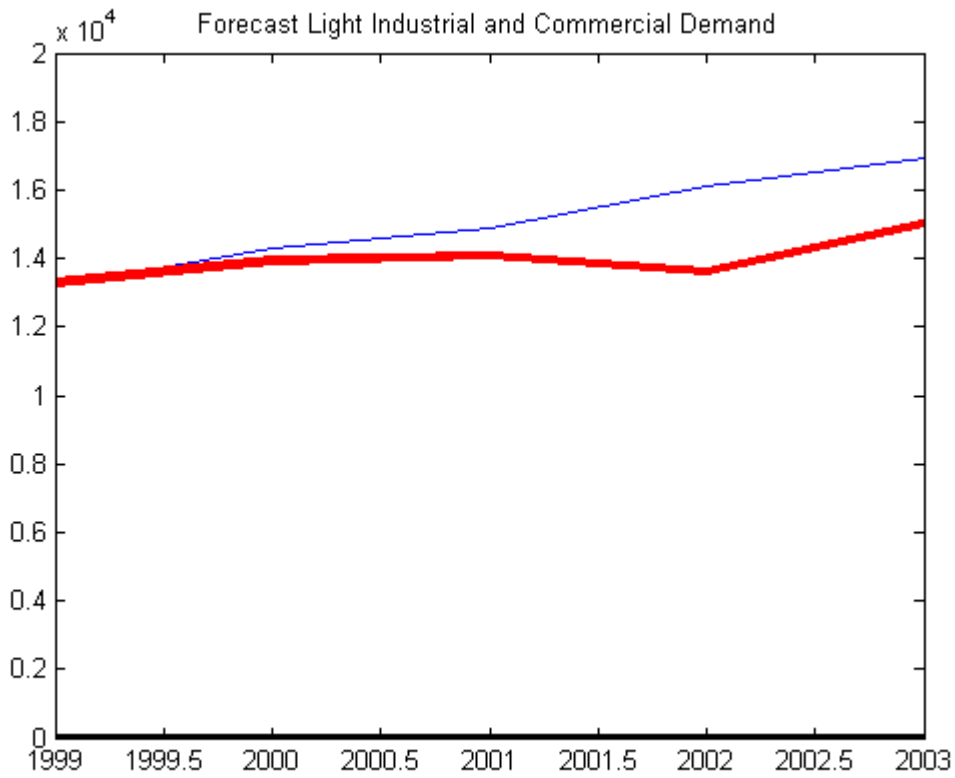
Two Stage Linear V5 – Input variation Monte Carlo 10000 runs



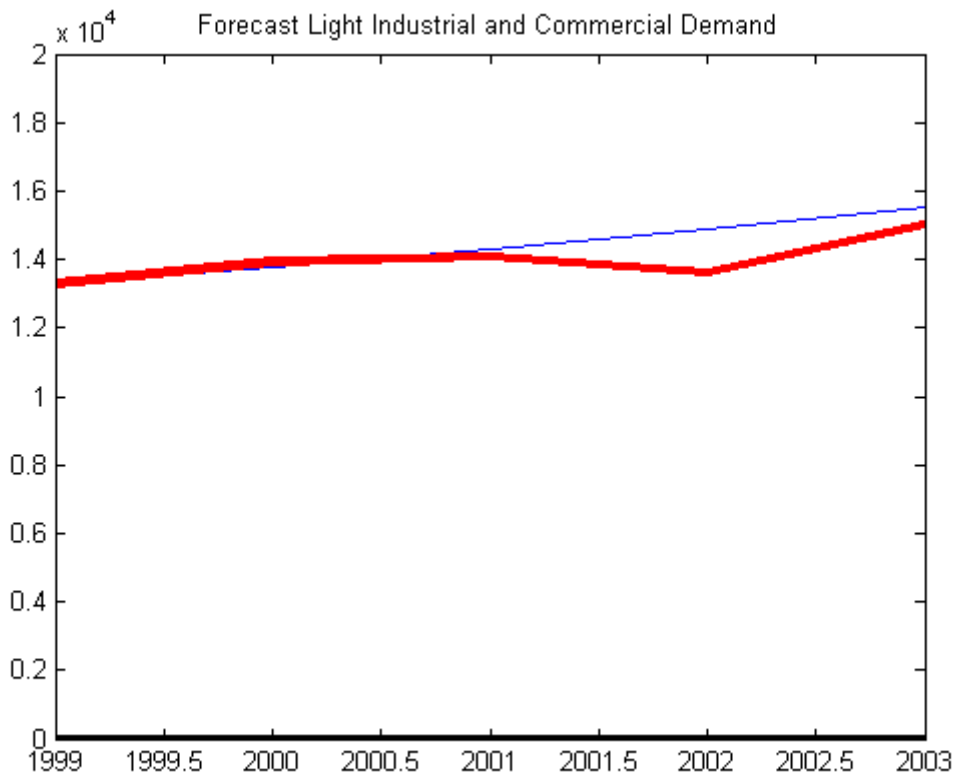
Two Stage Linear V3 (Transpower) – Truncated Forecast (Actual in Red)



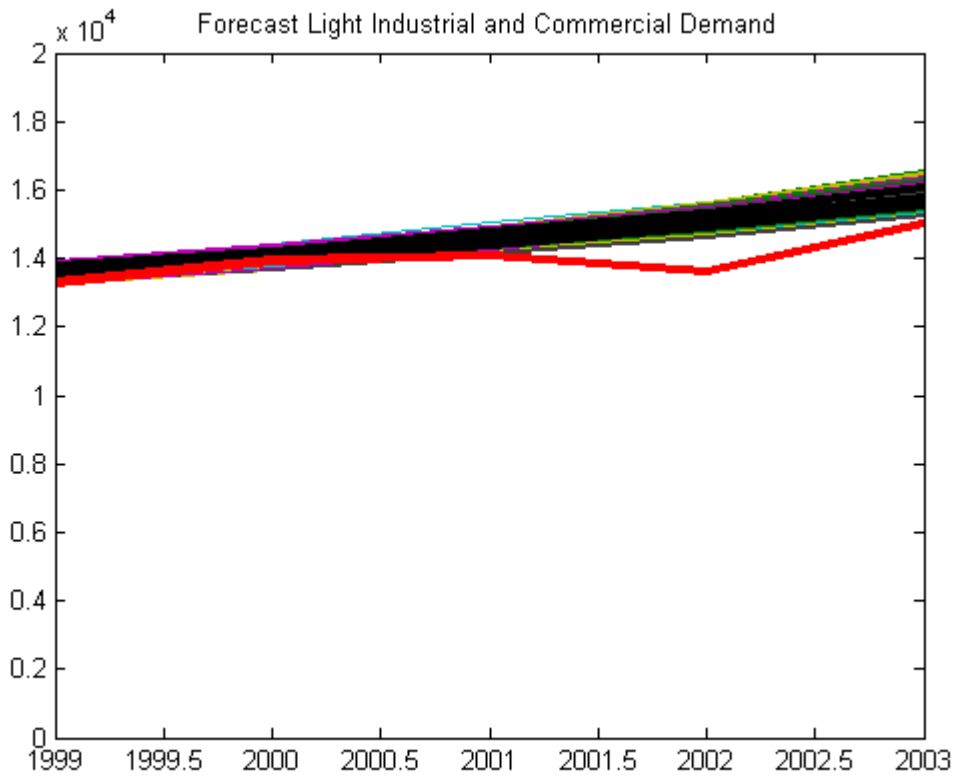
Two Stage Linear V1 – Truncated Forecast (Actual in Red)



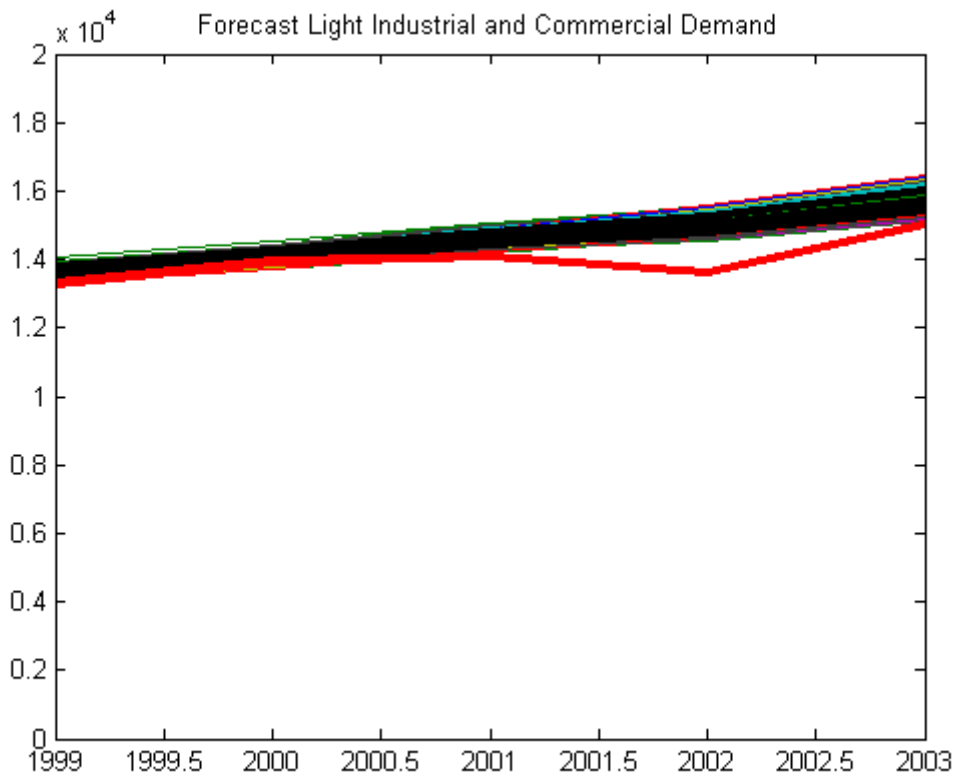
Single Stage Linear V2 – Truncated Forecast (Actual in Red)



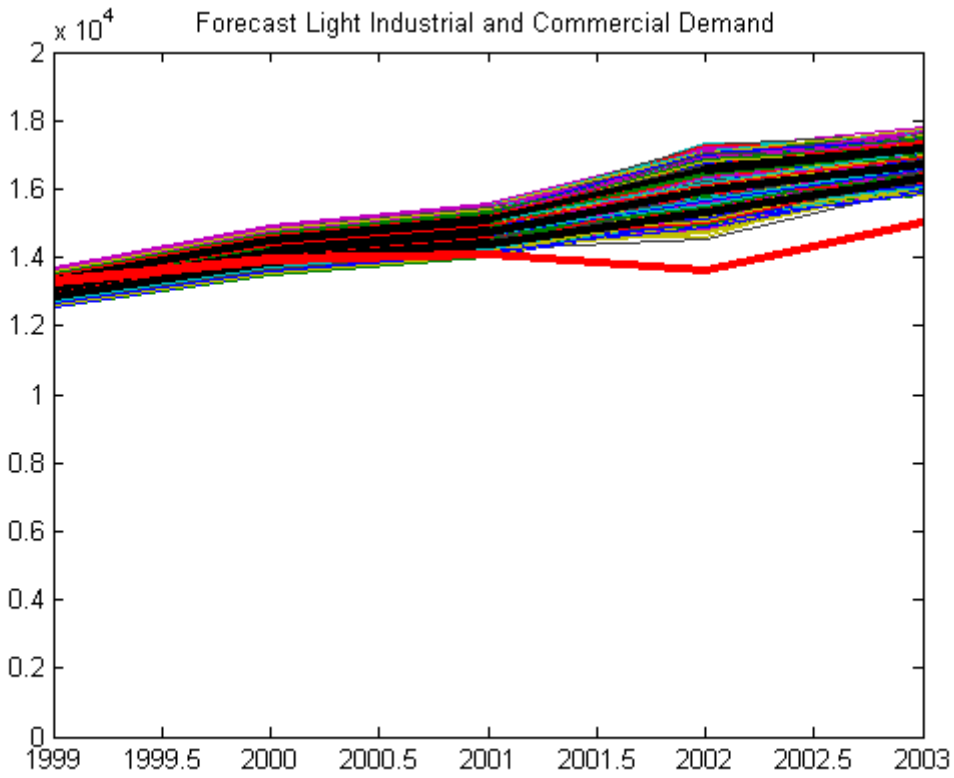
Two Stage Linear V5 – Truncated Forecast (Actual in Red)



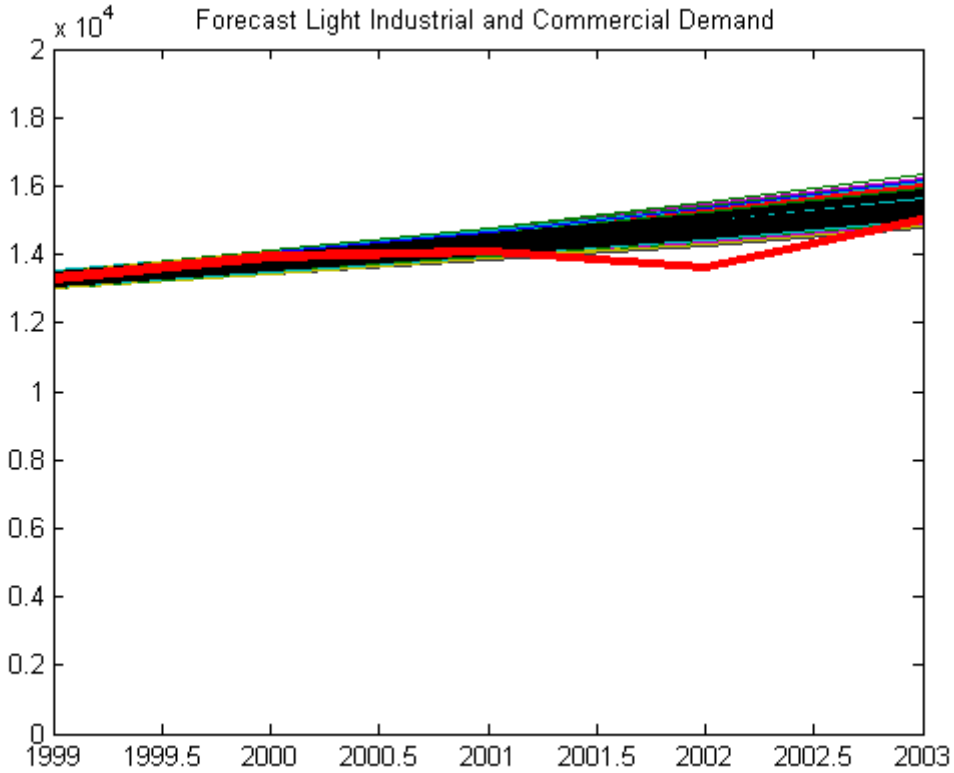
Two Stage Linear V3 (Transpower) - Monte Carlo 10000 runs (Actual in Red)



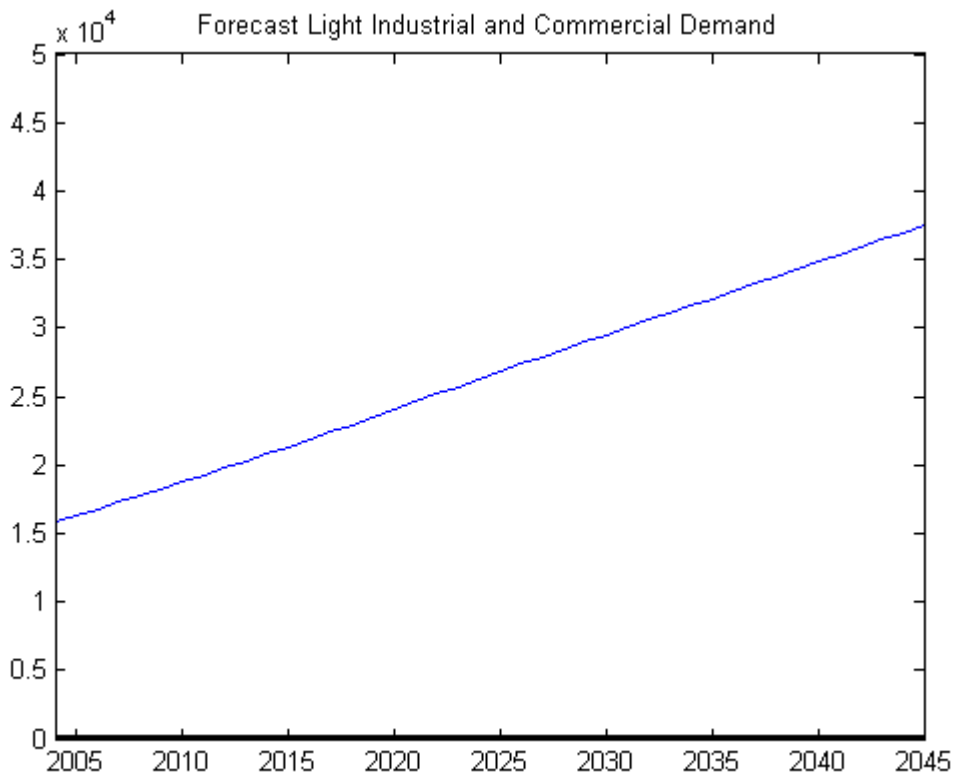
Two Stage Linear V1 - Monte Carlo 10000 runs (Actual in Red)



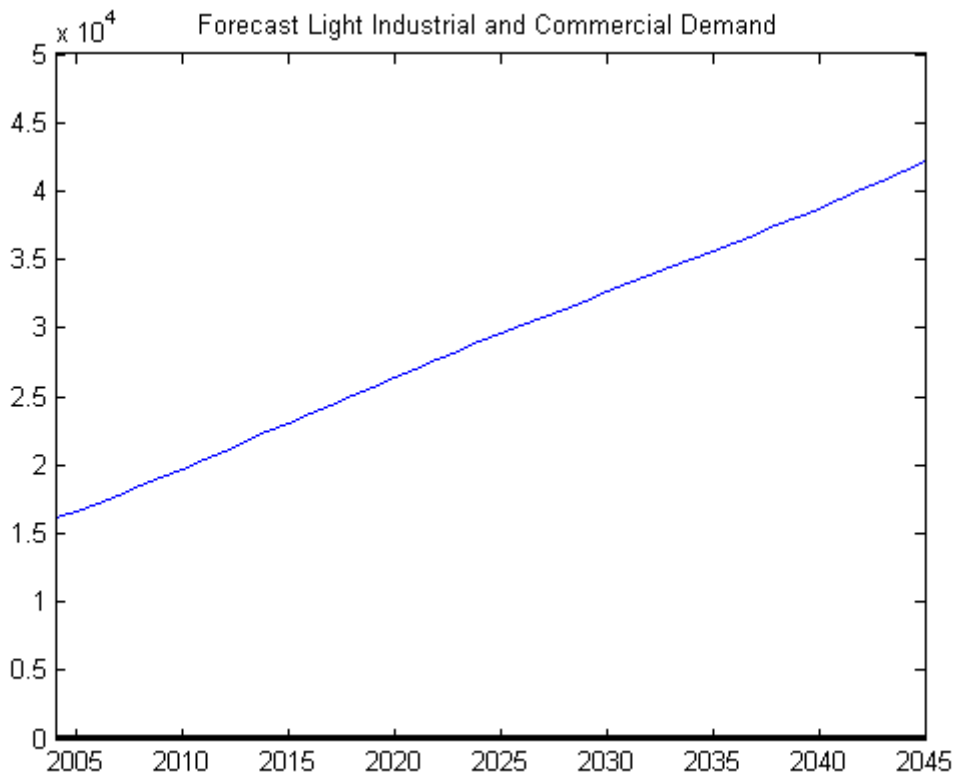
Single Stage Linear V2 - Monte Carlo 10000 runs (Actual in Red)



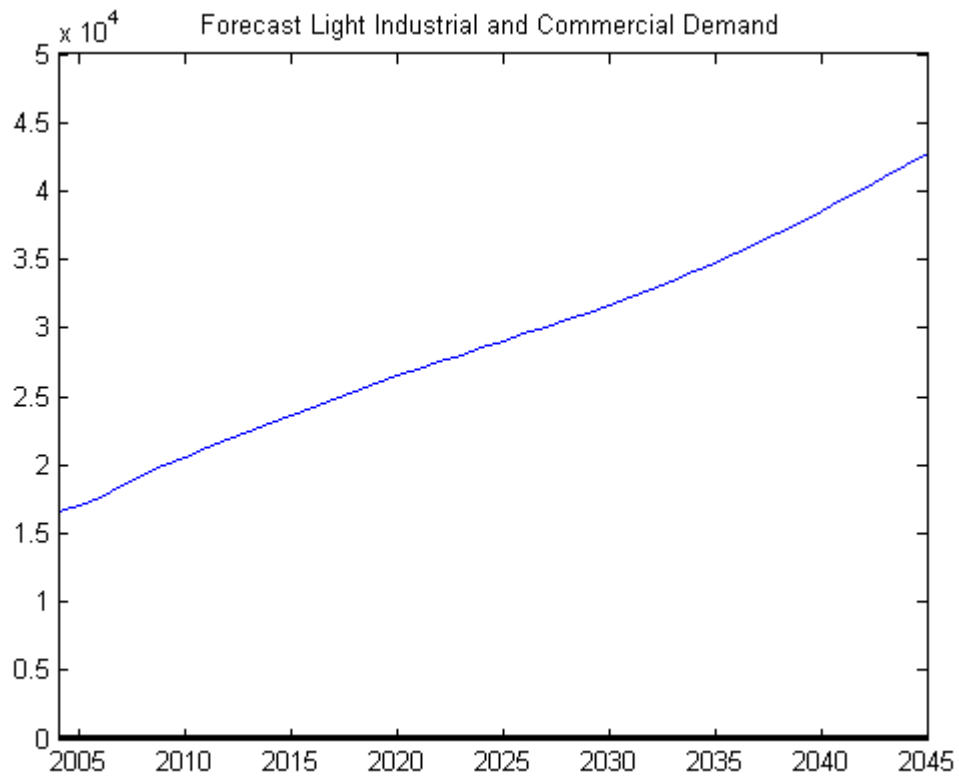
Two Stage Linear V5 - Monte Carlo 10000 runs (Actual in Red)



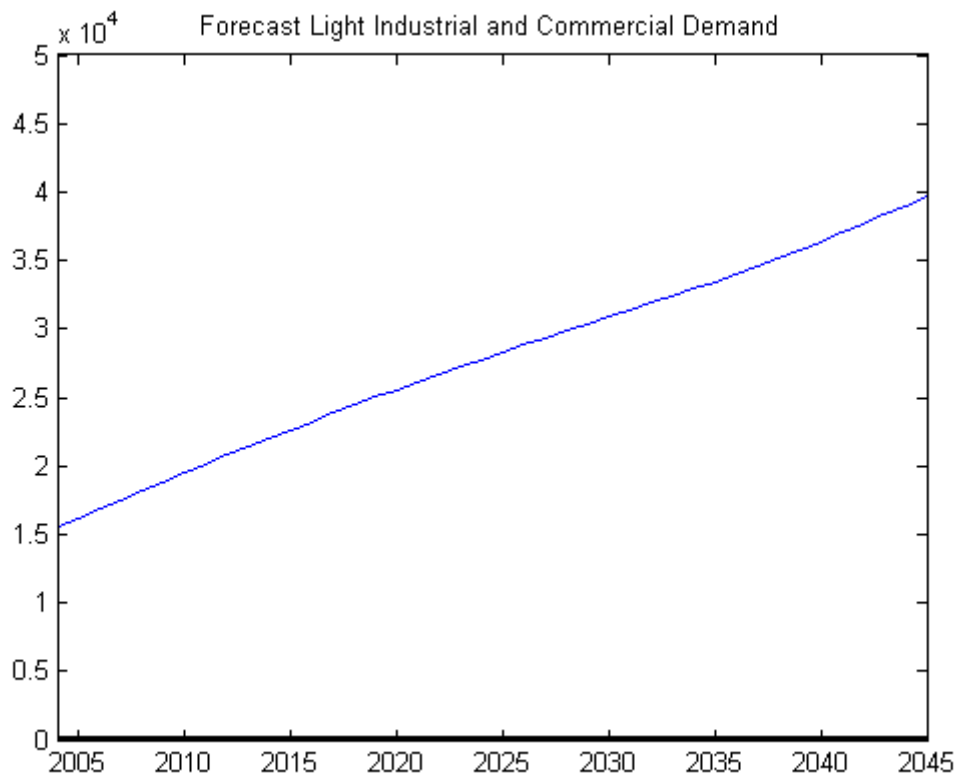
Two Stage Linear V3 (Transpower)



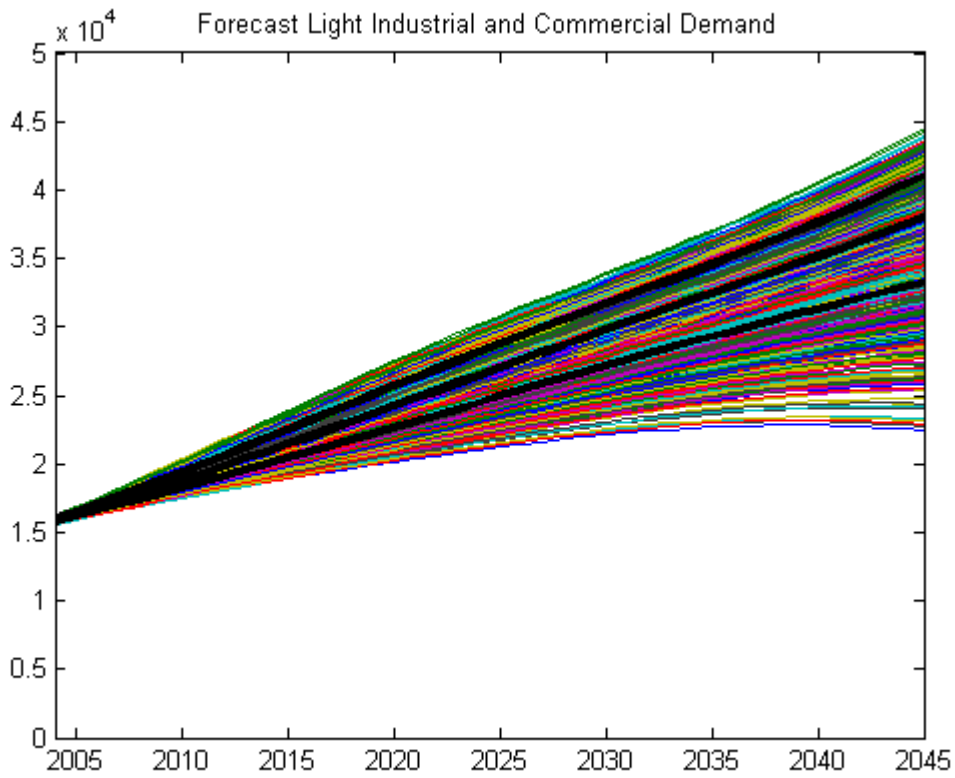
Two Stage Linear V1



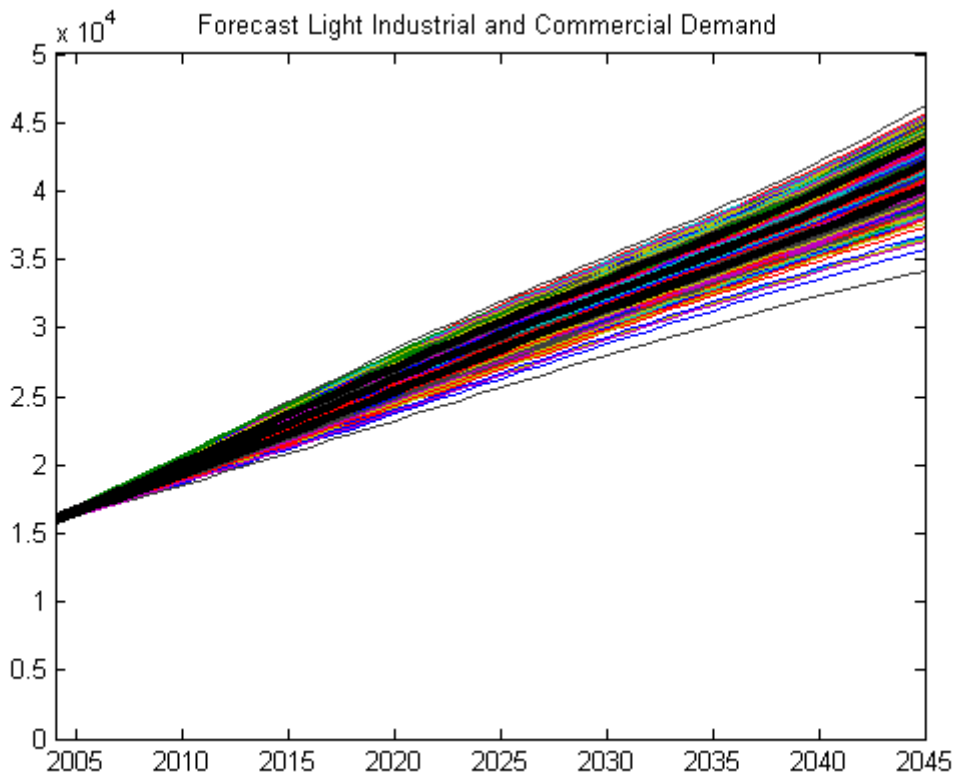
Single Stage Linear V2



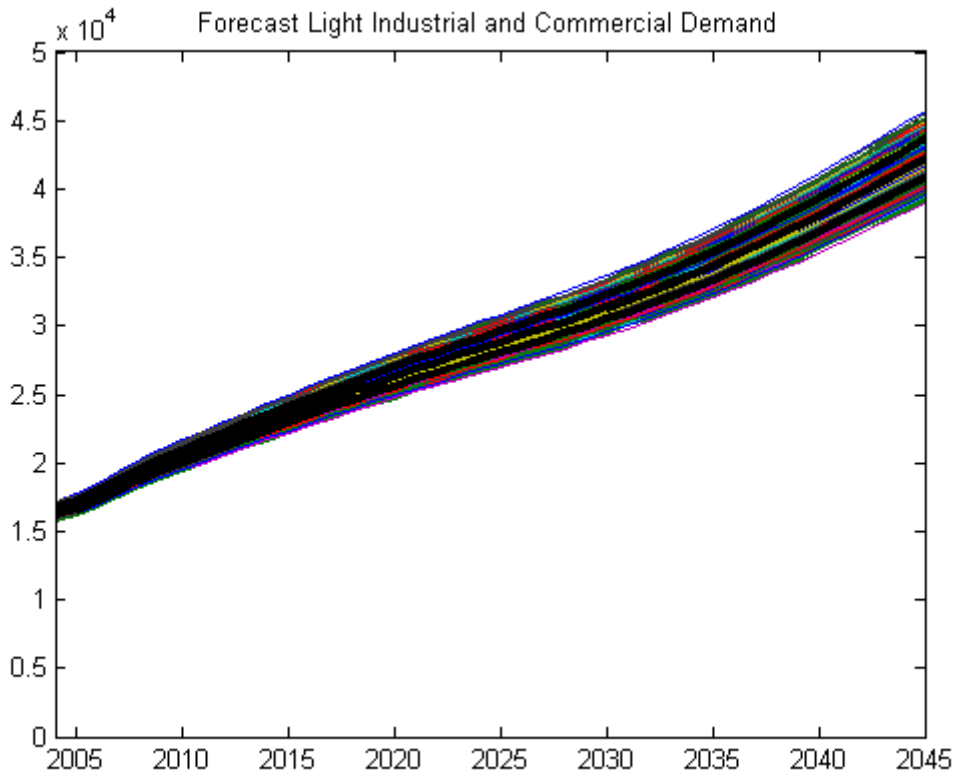
Two Stage Linear V5



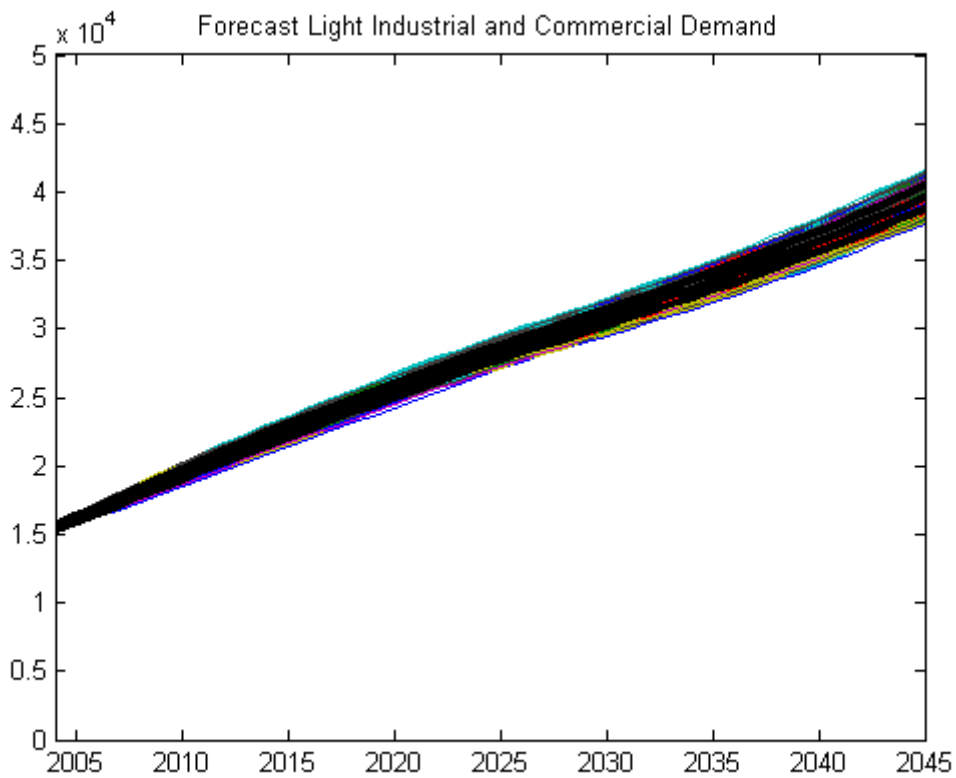
Two Stage Linear V3 (Transpower) - Monte Carlo 10000 runs 90 % conf.



Two Stage Linear V1 - Monte Carlo 10000 runs 90 % conf.



Single Stage Linear V2 - Monte Carlo 10000 runs 90 % conf.



Two Stage Linear V5 - Monte Carlo 10000 runs 90 % conf.